
Garth Heutel
Harvard Kennedy School
79 JFK Street, Mailbox 84
Cambridge, MA 02138
heutel@fas.harvard.edu

December 2008

JOB MARKET PAPER

Abstract

How should environmental policy respond to economic fluctuations caused by persistent productivity shocks? This paper answers that question by presenting and solving a dynamic representative agent real business cycle model that includes an externality from pollution, where pollution is a byproduct of production and can be reduced through abatement expenditures. The model is calibrated to consider emissions of carbon dioxide. I find that during economic expansions, it is optimal to allow emissions to increase. The effect of the higher price of abatement during periods of higher productivity outweighs the income effect of greater demand for clean air. I also model a decentralized economy, where government chooses an emissions tax or quantity restriction and firms and consumers respond. The optimal emissions tax rate and the optimal emissions quota are both procyclical: during recessions, the tax rate and the emissions quota both decrease.

JEL codes: Q58, E32, Q54

I would like to thank Don Fullerton, Bill Hogan, Bidisha Lahiri, Billy Pizer, Rob Stavins, Dan Tortorice, Richard Zeckhauser, and seminar participants at Harvard Kennedy School and the Southern Economics Association conference for helpful comments.
Environmental policy probably ought to respond to economic fluctuations and cycles, but the direction in which it ought to respond is unclear. During expansionary periods, for example, the economy is richer. Since a clean environment is a normal good, more of it will be demanded. On the other hand, expansionary periods involve increased production, which leads to increased pollution. Demand for cleaner air or water could come at a higher price during these periods of expansion. Additionally, how the damages from pollution depend on periodic fluctuations in pollution levels is potentially an important determinant of how environmental policy should respond to fluctuations. For some pollutants, including those that cause ground-level ozone, the marginal environmental damage curve is nonlinear within each period, thus an emissions increase in one period may lead to serious health consequences. For these pollutants, the costs of policy failing to respond to fluctuations can be high. For other pollutants, including carbon dioxide, where periodic fluctuations in emissions matter less than the cumulative stock of the pollutant, changing damages from pollution are unlikely to affect how policy responds to fluctuations.

Both the policy and academic literature are debating whether to "index" emissions policy, for example, by fixing a cap on emission as a multiple of a country's GDP—allowing the cap to be higher in periods of high productivity. Pizer (2005) argues that intensity targets are preferable to absolute targets for a number of reasons, such as accommodating growth in developing countries. The Bush administration has argued for setting such intensity targets. Newell and Pizer (2006) compare indexed policies to price and quantity policies and find that indexed policies may be preferable in some circumstances. By calibrating their model for the largest greenhouse gas emitting nations, they find that indexed quantities dominate fixed quantities for about half of the nations in this group. Similar results are found in Jotzo and Pezzey (2007) and Sue Wing et. al. (forthcoming). A related topic under much discussion is that of cost-containment, or developing a policy to ensure that costs do not become unreasonably high due to uncertainty, unanticipated shocks, or other reasons.

The purpose of this paper is to investigate how environmental policy would optimally respond to autocorrelated fluctuations in economic productivity. To formalize intuition, I first develop a static theoretical model of the optimal allocation of resources between production and abatement, where both factor productivity and wealth are subject to separate shocks. The solution analytically presents the trade-offs associated with productivity shocks. Then, I extend
the theoretical model to include dynamics, where productivity shocks can be persistent and where pollution and capital are stock variables. The model is calibrated to the US economy and to damages from carbon dioxide, the leading greenhouse gas, and I numerically solve for the dynamic optimal response of policy to shocks. I also model a decentralized economy, where firms and consumers optimize in response to a government policy of pollution taxes or quantity constraints, potentially with information asymmetry.

The contributions of this work to the literature are three-fold. First, many earlier studies are static, and I extend this model into a dynamic setting. The extent to which pollution is a stock rather than a flow affects the impact of fluctuations on optimal policy levels. Likewise, a dynamic model is necessary to capture effects of autocorrelated productivity shocks and stock variables like capital. Second, earlier papers are focused on uncertainty, where the indexed policy is advantageous because it mitigates regulator’s uncertainty about marginal abatement costs. For example, in Newell and Pizer (2006), the index (e.g. GDP) is a signal to policymakers of the uncertain abatement costs. Without uncertainty in those models, indexing offers no advantage. In the basic model here, policy may be indexed to GDP even without uncertainty in abatement costs. Because GDP is a function of productivity shocks, those shocks influence optimal policy through income and price effects. Furthermore, indexing to GDP need not be linear, and the optimal index is likely to be a nonlinear function of GDP. Additionally, the model here is extended to include asymmetric uncertainty, when firms but not policymakers observe technology shocks.

A third contribution is that, while most earlier papers use a very specific structure of uncertainty and abatement costs and benefits, here I depart from that specification. Instead of imposing a reduced form quadratic cost function and another reduced form quadratic benefit function, I begin with a standard dynamic stochastic general equilibrium (DSGE) model of real business cycles. To this I add an externality that comes from an accumulated stock of pollution. The stochastic element of the model does not arise from shocks to abatement costs; rather, it comes from total factor productivity shocks. This is an important distinction, because it creates a price and an income effect that can counter each other, such that the response of optimal policy is

---

1 See Fell et. al. (2008), Newell and Pizer (2003), Hoel and Karp (2002), Pizer (2002), and Kolstad (1996) for examples of papers with a dynamic model and emissions as a stock variable.

2 Argentina in 1999 proposed a greenhouse gas emissions target that was proportional to the square root of GDP (Aldy 2004, footnote 38).
not apparent, even in the absence of uncertainty over the value of the shock. Furthermore, this choice of modeling is advantageous in that I can draw upon a large prior real business cycle (RBC) literature to calibrate the model and evaluate its predictions. The reduced form shocks to abatement costs and benefits present in other models are likely due to productivity shocks, and here they are modeled directly as such.

The paper begins with an empirical examination of how carbon dioxide emissions in the United States respond to cyclical fluctuations in GDP. I estimate the elasticity of emissions with respect to GDP. The expected sign of this elasticity is positive, but its magnitude has not been measured. Using both monthly data on GDP and carbon dioxide (CO₂) emissions and annual state-level data, I find that emissions are significantly procyclical with an elasticity between 0.5 and 0.9. Thus, emissions are inelastic with respect to output. This result is robust to a number of empirical specifications. The purpose of the empirical section is twofold. First, a measurement of this elasticity is interesting in and of itself (during a recession of a specified magnitude, by how much do we expect emissions to drop?) Second, the estimated elasticity is used as a parameter in the calibrated model.

In the static model, I find that optimal environmental policy does indeed vary with productivity and income shocks in ways that exemplify both price and income effects. If a positive shock is purely to wealth and has no effect on marginal returns to investment, then optimal policy is unambiguously to decrease emissions. The income effect leads the economy to demand a cleaner environment. With a positive shock to productivity that increases both wealth and the rate of return on investment, optimal policy faces a trade-off. The income effect leads to higher demand for a clean environment and thus lower emissions. However, those lower emissions come at a higher price: since capital is more productive, abatement is relatively costlier. Which effect dominates depends on the parameterization of production and utility functions.

These effects are present in the dynamic model, but analytical solutions are not possible in the general case. After calibrating the dynamic model and solving it, I find that the optimal policy response to an economic expansion is to increase emissions, because the price effect dominates the income effect. In fact, a policy that pegs emissions to GDP is a good approximation of optimal policy. This qualitative result is robust to sensitivity analysis over several parameters. The paths of some other variables, including emissions, do qualitatively
differ under different parameter values. Simulating the decentralized economy shows that both the optimal emissions tax rate and the optimal emissions quota are procyclical: they increase during an expansion. Optimal policy thus dampens the procyclicality of emissions: emissions rise during booms and fall during busts, but not by as much as they would without optimal policy. An emissions quota policy is strengthened during recessions (the quota is reduced), while an emissions tax policy is weakened (the tax rate is reduced). This is perhaps a political advantage of taxes over quotas, if it is difficult to strengthen environmental policy during recessions.

Several previous studies have considered environmental policy in the context of real business cycles or other economic fluctuations. Chay and Greenstone (2003) estimate the effect of air pollution on infant mortality, in a quasi-experimental design where the 1981-1982 recession affects emissions of total suspended particulates differentially across counties. Their concern is the effect of emissions on health, not optimal policy in the presence of business cycles. Brannlund and Lofgren (1996) analyze the impact of environmental policy when emissions are subject to random fluctuations, not total factor productivity. Strand (1995) develops a model with environmental policy that focuses on worker moral hazard with business cycles operating through stochastic changes in output prices. The paper most similar to this paper is Bouman et. al. (2000), which develops a model to find the optimal time to invest in abatement, given business cycles. They find that the best time to implement cleaner production technologies is during downturns. Their model is one in which the social planner optimally chooses whether to use clean or dirty production technologies in each period. Their model differs from RBC models in that business cycles are not driven by productivity shocks, but by preference shocks. They also estimate if industries do indeed optimally invest in abatement during downturns, using data on abatement investment from Germany, the Netherlands, and the United States. They find so for only 25% of sectors.

In the next section, I present some dynamic descriptive statistics regarding the relationship between business cycles and carbon dioxide emissions in the US. Section II presents a simple static model of optimal policy under productivity and income shocks, to make concrete the intuitions regarding when optimal abatement ought to occur. Section III presents a dynamic model of a social planner's problem, which is calibrated in Section IV. The model is solved and simulation results are presented in Section V. Section VI presents a decentralized
version of the dynamic model, where competitive firms and utility maximizing consumers react to government policy that can take the form of an emissions tax or a quantity constraint. Finally, Section VII concludes.

I. Descriptive Statistics

I investigate the relationship between business cycles and emissions using various sources of data on emissions at the monthly or quarterly level. While emissions data are widely available at the annual level for many different pollutants, data at more frequent intervals are scarcer. Blasing et. al. (2004a) provide estimates of monthly emissions of carbon dioxide from fossil fuel combustion in the United States from 1981-2003. These estimates are based on reported fuel consumption from the Energy Information Administration in the Monthly Energy Review for several types of fossil fuels. Each type of fuel is converted into its associated level of carbon dioxide emissions, based on fuel-specific carbon dioxide emissions factors from the EPA. Figure 1 plots the unadjusted monthly levels of nationwide emissions, in teragrams of carbon (1 teragram = 1 trillion grams). Both an increasing trend and a seasonal cycle are evident. Carbon emissions have a large peak during the winter months, due to an increase in natural gas consumption for heating, and a smaller peak in the summer months, from increased coal consumption for electricity use.

Because of the strong seasonal component, I seasonally adjust the emissions data to isolate trends and cyclical components. Data on GDP used to identify business cycles are routinely seasonally adjusted before being released. The routine used by the Census Bureau to seasonally adjust their data series is X-12-ARIMA, software that performs an autoregressive integrated moving average (ARIMA) time series regression. X-12-ARIMA has a large number of options available for specifying the regression and seasonal adjustment, including routines for evaluating and accommodating outliers, selecting optimal lag lengths for AR and MA processes, and adjusting for length-of-period. The routine can thus be criticized as a "black box."

However, the seasonally adjusted data that come from the program are quite robust to the various specifications in X-12-ARIMA, and are furthermore robust to simpler methods of seasonal adjustment, such as calculating a centered moving average or simply running a regression with

---

4 The software can be downloaded, and support information can be found at: http://www.census.gov/srd/www/x12a/.
monthly indicator variables. The seasonally adjusted time series of emissions using any of these methods has a correlation coefficient with any other seasonally adjusted series of at least 0.99. All results presented hereafter use the seasonally adjusted data from X-12-ARIMA, where the optimal seasonal ARIMA model is found to be (1 1 1)×(0 1 1) and outliers are not eliminated. Results are qualitatively robust to other methods of seasonal adjustment, and regression results using these series are available upon request.

The emissions data are seasonally adjusted at the monthly level. Figure 2 plots the seasonally adjusted monthly emissions data, along with monthly real GDP, provided by the Bureau of Economic Analysis and seasonally adjusted also. Both series are normalized so that the starting value is 1 (in January 1981).

Over the period of analysis, both GDP and carbon emissions grew, but at different rates, so that the carbon intensity of the economy declined. GDP increased by a factor of 2, while carbon emissions increased only 25%. Business cycle effects can also clearly be seen in this figure; recessions in the early 1980s, early 1990s, and early 2000s are reflected in the GDP curve. Concurrently with these recessions, carbon emissions appear to drop off. Similarly, when GDP is rising at a fast rate, carbon emissions appear to do the same. This happens during the expansionary period of the mid- to late-1990s.

While basic patterns can be seen from eyeballing Figure 2, a more thorough method of identifying cycles in output or in emissions is available. The Hodrick-Prescott (HP) filter is a commonly used method to detrend time series, separately identifying the trend component from the cyclical component. The HP filter does so by finding the smooth path that minimizes a function of deviations from that path. I aggregate the monthly emissions and GDP data to the quarterly level as is commonly used in business cycle literature. I take the natural log of the values in the time series. Thus, the difference between subsequent observations in one series approximates a growth rate, and the values of the cyclical components of the series represent proportional deviations from the trend. For the smoothness parameter $\lambda$, I use the value 1600, which is typically chosen for quarterly data.

---


6 For a time series of values $y_t$, the smooth trend components $s_t$ are those that minimize:

$$\sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=1}^{T} [(s_{t+1} - s_t) + (s_t - s_{t-1})]^2 ,$$

where $\lambda$ is a positive parameter.
The cyclical component of the detrended data is presented in Figure 3. The patterns in the GDP curve correspond to recessions: troughs in the early 1980s, the early 1990s, and the early 2000s. For the emissions curve, some of these troughs can be clearly identified concurrently, especially in the early 1980s and early 1990s recessions. The correlation between the two series seems to dissipate starting in the late 1990s, though a drop in GDP in 2000 is accompanied by one in emissions as well. Overall, it appears that emissions are more variable than GDP, and that magnitudes of deviations in emissions are larger than those of GDP.

The variation of and correlations between the two time series can be measured. The standard deviation of cyclical GDP is 1.31%, while the standard deviation of cyclical carbon emissions is 2.04%, so emissions are indeed more variable than GDP. The correlation coefficient between the two time series is 0.5627, with a p-value less than 0.0001. The two time series are strongly correlated. Unsurprisingly, when output increases, so do carbon emissions.

These data can also be analyzed using time-series analysis. Unconditional correlations show that periods of higher GDP (in deviations from trend) tend to occur with periods of higher CO₂ emissions. But a question of interest and relevance is the following: what is the magnitude of the relationship between CO₂ emissions and GDP? Emissions may be elastic with respect to GDP, in which case an increase in GDP will be associated with an increase in CO₂ of a greater magnitude. Alternatively, CO₂ emissions may be inelastic with respect to GDP. Table 1 presents regression results to identify the magnitude of the relationship between CO₂ emissions and GDP. In column 1, I present regression results from a seasonal ARIMA(1,1,1)×(0,1,1)_{12} regression of the log of emissions on the log of GDP. The optimal lag lengths and differencing were determined by minimizing the Akaike Information Criterion (AIC). The multiplicative seasonal ARMA component accounts for the fact the CO₂ data used in Column 1 are not seasonally adjusted, and the differencing accounts for trends in both series. Both series are nonstationary; an augmented Dickey-Fuller test finds strong evidence for unit roots in each series (for log of GDP, the test statistic is −0.033 with a p-value of 0.9557; for log of CO₂ emissions the test statistic is −0.708 with a p-value of 0.8446). Both series are also integrated of order one. The regression results show that CO₂ emissions are inelastic with respect to GDP, with a coefficient of 0.758.

---

7 As it appears in the graph, the correlation between the two series is higher in the first half of the period (0.714) than in the second half (0.164), though still significantly positive in both.
An alternative method of dealing with the seasonal component of the emissions data, besides estimating it through a seasonal ARIMA regression, is to perform the regression on the emissions data that have already been seasonally adjusted. This is done is Column 2, and the coefficient on GDP is almost identical to that in Column 1 (in this regression, minimizing the AIC dictates an ARIMA (1,1,2) regression).

Both of these regressions have used first differencing to eliminate the trend from both the GDP series and the emissions series. Alternatively, a number of other filters are available to better identify the trend versus the cyclical or irregular components of the series. The most commonly used filter in the business cycle literature if the Hodrick-Prescott filter, as described above. Column 3 presents regression results where the dependent variable is the deviation from trend in the log of CO₂ emissions, determined by the HP filter, and the independent variable is the deviation from trend in the log of GDP. The smoothing parameter $\lambda$ is set at 129,600 since the data are monthly. A least squares regression is performed, allowing for a Newey-West specification of the error term. The coefficient is consistent with the results from the ARIMA regressions; CO₂ emissions are inelastic with respect to GDP.

The HP filter is not the only method for detrending time series. Three additional filters are applied to the data, and regression results on the detrended data are presented in Columns 4-6. The Baxter-King filter is a bandpass moving average filter that filters out the trend as well as higher frequency components. The minimum and maximum periodicities are set at 18 and 96 months, respectively. The Christiano-Fitzgerald filter is a random walk filter, and the same minimum and maximum periodicities are used as in the Baxter-King filter. Finally, the digital Butterworth filter is a rational square wave filter. In regressions presented in Columns 4-6, I detrend the GDP and CO₂ emissions series with each of the three filters and run the same regression as in Column 3.

The purpose of these additional regressions is not to make any claim about which filtering method is preferable. Rather, I seek to demonstrate that the key result found in the first three columns of Table 1 is robust to a wide range of filtering methods. In fact, this is what I find, as can be seen from the regression coefficients in Table 1. The coefficient on the deviation from trend in GDP is consistently positive and between 0.5 and 0.9. The result that emissions are procyclical but inelastic with respect to GDP are thus quite robust. In addition, analogous regressions on data that are aggregated to the quarterly level yield results almost identical to
those presented in Table 1. The results are also robust to varying the lag lengths in the ARIMA regressions and varying the smoothing parameter(s) in each of the filters, and all regressions are robust to including world oil spot prices and seasonally adjusted temperatures as exogenous regressors.

The carbon emissions data analyzed above are at the national level. Blasing et. al. (2004b) also provide estimates of state-level carbon emissions from 1960-2001. The state-level data are annual, not monthly, limiting the extent to which cyclical behavior can be studied. However, when emissions are disaggregating by state, between-state heterogeneity in output and emissions can be used to analyze further the relationship between these two variables. Table 2 presents regression results from a model where state-level logs of carbon emissions are regressed on state-level logs of real GDP. State-level GDP is available from 1963 on from the Bureau of Economic Analysis. In column 1, a simple OLS regression of the log of emissions on the log of GDP shows a positive correlation between the two. Of course, both series are trending upwards, which could bias this coefficient. Therefore, in column 2, I include a set of state and year fixed effects. The coefficient on the log of GDP is smaller but still significantly positive. Finally, in column 3 I account for autoregressive error terms. This adjustment cuts the magnitude of the coefficient about in half, but it remains significant. Thus, the state-level (though annual) data provides more compelling evidence of the observation made from examining the national-level data: carbon emissions are higher in places and periods of higher output.

Though the purpose of this study is to analyze optimal carbon policy, one could also perform similar analyses on the cyclical behavior of other pollutants. I have also used monthly data on emissions of sulfur dioxide (SO$_2$) and nitrogen oxides (NO$_X$) from electric power plants from the EPA’s Clean Air Markets program. In fact, for those two pollutants, the patterns in both the trend and the cyclical components are quite different than for CO$_2$. Both SO$_2$ and NO$_X$ are trending downwards, not upwards, from 1997-2006. This reflects the fact that these pollutants, unlike CO$_2$, are regulated by the Clean Air Act. For SO$_2$, during this period, power plants faced a cap-and-trade scheme, where the total allowable emissions from the industry steadily decreased each year. NO$_X$ standards also became more stringent over time. Additionally, the correlation between cyclical components of GDP and cyclical component of these pollutants (after they are

---

detrended in the same manner as described above) is positive but very small and not statistically significant. This suggests that when producers are constrained by regulation in how much they can emit, they are less able to adjust emissions to aggregate economic fluctuations, and thus emissions of these pollutants are less correlated with output.

II. Static Model

Before considering a dynamic model of environmental policy that allows for the accumulation of capital and pollution, I present a static model where decisions on consumption and emissions do not carry over beyond the current period. The advantage of presenting this simpler model is the intuition it provides regarding the tradeoffs inherent in abatement choice. These tradeoffs will be present in the dynamic model, though the use of a static environment allows me to find analytical solutions for optimal policy, and these solutions conveniently demonstrate the tradeoffs. To show how income effects and price effects differentially influence policy decisions, I include in this static model both a productivity shock and an income shock.

Consider a representative agent economy with a utility function \( U = y - d(e) \), where \( y \) is consumption, \( e \) is emissions, and \( d(e) \) is the damage function from emissions. Let \( d(e) \) be increasing and convex. The representative agent has access to a production function \( f(ak) \), where \( k \) represents the inputs used in production (capital), and \( a \) is an exogenous shock to productivity. Let \( f'(ak) > 0 \) and \( f''(ak) \leq 0 \), so that the marginal productivity of capital is positive and non-increasing. The agent is endowed with income \( b \). The income can be distributed into two uses: production and abatement. The resource constraint is thus \( k + z \leq b \), where \( z \) is the amount of abatement. This economy has two sources of exogenous variation: \( a \) is a productivity shock, and \( b \) is an income shock. The level of emissions is determined by both production and abatement: \( e = g(z)f(ak) \), where \( g(z) \) is an abatement function. Suppose that \( g(z) \) is decreasing and convex, so that emissions decrease with more abatement but at a decreasing rate. Finally, consumption is just equal to output: \( y = f(ak) \).

The representative agent’s problem is to choose \( k \) and \( z \) to maximize utility \( f(ak) - d(e) \), subject to the resource constraint \( k + z \leq b \). Given that the resource constraint must bind, the problem can be written as the choice over just \( k \), and the corresponding first order condition is

\[
af'(ak) - d'(g(b-k)f(ak)) [-g'(b-k)f(ak) + g(b-k)af'(ak)] = 0.
\]
The first order condition ensures that the marginal benefit of an additional unit of capital, \( af'(ak) \), equals the marginal cost, which is the increased emissions resulting from the marginal switch from abatement to capital, given in the second term of the above equation.

The first order condition can be used to consider comparative statics. Specifically, how is the choice between capital \( k \) and abatement \( z \) dependent on the productivity shock \( a \) and the income shock \( b \)? For simplicity, assume that the production function \( f \) is linear. Appendix A1 uses the implicit function theorem on the first order condition to provide the answer to this question. The relationship between the shocks and the optimal allocation are consistent with the counteracting income and price effects described above.

First, consider the effect of the income shock, \( b \). This shock is purely to the amount of resources available, and not to productivity, and thus it creates only an income effect. The Appendix shows that both \( dk/db \) and \( dz/db \) are positive. When income increases, the optimal response is to distribute additional resources to both production and abatement, given that consumption and a clean environment are both normal goods. This response results in unambiguously increased consumption, but the effect on emissions is not yet clear. However, it can be further shown that \( de/db < 0 \). Thus, a positive income shock has an unambiguous negative effect on emissions.

Next, consider the effect of the productivity shock, \( a \). This shock provides both an income and a price effect. When \( a \) is high, the representative agent has more a productive technology and is thus richer. This contributes to increased demand for production and abatement. However, a high \( a \) also means that capital is more productive and therefore also more polluting; the same choice of capital \( k \) comes with higher output and emissions. The marginal increase in emissions from an increase in capital expenditure is now higher. The increased emissions caused by an increase in capital pushes the agent to choose less capital. This effect counters the demand for more capital. The net effect can be written as proportional to an expression that includes both of these opposing effects:

\[
\frac{dk}{da} \propto 1 - (-g'k + g)(d''gk) - gka).
\]

Each expression in parentheses is positive. The positive 1 at the start of the expression represents the income effect associated with a technology shock. The rest of the expression (including the minus sign) is negative, and it represents the price effect (the relevant price is the price of
abatement, which increases with positive productivity shocks). The effect on abatement from a change in productivity $dz/da$ is simply equal to $-dk/da$. Conflicting price and income effects also operate on the optimal level of emissions:

$$\frac{de}{da} \propto -g'k + g + d'gk^2g'' - d'(g^2 + g'^2k^2).$$

The expression for $de/da$ is grouped into two sets of terms. The negative terms represent an income effect, and the positive terms represent a price effect.

To determine which of these offsetting effects dominates, more information on the functional forms in the model is necessary. I present these results from the static model to provide some analytic structure to the countervailing intuitions regarding how optimal policy responds to technology shocks. Next, I expand the model into a dynamic setting, calibrate it, and solve for optimal policy. While the dynamic model does not allow for the same comparative static analysis as is presented here, solving the model with the calibrated parameters allows determination of which effect dominates, and thus how optimal policy should respond both qualitatively and quantitatively to economic fluctuations.

III. Dynamic Model – Centralized Economy

While the static model presented above gives intuition regarding two conflicting effects that push for more and less abatement during periods of high productivity, it fails to account for dynamic considerations. At least three such considerations are built into the following dynamic model, corresponding to three state variables. First, the productivity shock $a$ is likely to be autocorrelated. This autocorrelation of factor productivity shocks is in fact what drives most RBC models. Thus, a high value of the shock in one period also serves as a signal about the likely shock in subsequent periods. Second, capital $k$ in the economy is a stock good. Choosing to save more during one period leads to more resources available for consumption or abatement in subsequent periods, after depreciation. The decision over the allocation of resources between consumption and abatement in the static model is amended to include the option of investment as well. Third, the damages from pollution may come not just from emissions in the current period but from the total stock of emissions. This stock is a function of current and past levels of emissions, subject to a depreciation function that is unique to the chemistry of the pollutant in question. For example, CO$_2$ is a stock pollutant with a half-life in
the atmosphere of several decades. On the other hand, ground-level SO$_2$ has a half-life of only a few days, so for the purposes of a business cycle model where each period is one quarter, it can be considered purely as a flow. Furthermore, for a global pollutant like CO$_2$, the stock is determined not just by domestic emissions but also by emissions from the rest of the world.

In this section I consider a centralized economy; that is, I model the social planner's problem. In the standard RBC model, this choice of modeling is justified by the fact that the economy lacks externalities and thus satisfies the first fundamental theorem of welfare economics. A competitive equilibrium is also a Pareto-efficient one, and by modeling a social planner's problem, the equilibrium is easier to solve. Here, the first fundamental theorem is not generally expected to be satisfied, because of the externalities imposed by pollution. In this section, I present a model of optimal policy, in the case where a central authority can select investment, abatement, and consumption for the representative consumer. Later, I model a decentralized economy, where government can attempt to fix the inefficiencies associated with the pollution externality through a policy, such as an emissions tax or tradable permits. The model does not explicitly model growth in technology, consumption, or output. The assumption that technology is invariant makes the model stationary and allows analysis of temporary fluctuations about the steady state. As with other RBC models, the model here can be derived from a model that includes a constant growth rate. The variables in the model with growth can all be divided by the value of the technology parameter to generate a model identical to the one here.\textsuperscript{10} Thus, deviations from the steady state in this model actually represent deviations from steady state growth.

Consider a representative agent with access to the same production technology as in the static model above: $y_t = f(a_t, k_{t-1})$. Variables are time-subscripted because the model is dynamic; output in period $t$ $y_t$ is a function of a current productivity shock $a_t$ and capital carried over from last period, $k_{t-1}$, thus providing a "time to build." In each period, the agent chooses quantities of consumption $c_t$, abatement $z_t$, and investment $i_t$, subject to the resource constraint determined by current production: $c_t + z_t + i_t \leq f(a_t, k_{t-1})$. Note that just one input is used in production. Labor is not included in the model, since this paper is not concerned with employment fluctuations. While the static model included a productivity shock $a$ as well as an

\textsuperscript{10} See King et. al. (1988).
income shock \( b \), this model only includes the productivity shock.\(^{11}\) Income is not exogenous; it is determined by the agent's choice of investment in the previous period along with the current productivity. Furthermore, the last section shows that the productivity shock alone creates both income and price effects.

The utility function is over consumption and the stock of pollution, \( x_t \), and is more general than in the static model: \( U(c_t, x_t) \). Pollution is a stock good with a linear decay rate equal to \( \eta: x_t = \eta x_{t-1} + e_t + e_{row_t} \), where \( e_t \) is current-period domestic emissions and \( e_{row_t} \) is current-period emissions from the rest of the world. The policy-maker cannot choose the level of emissions from the rest of the world but can choose the level of domestic emissions. As in the static model, domestic emissions are a function of total production \( y_t \) and abatement, although unlike in the static model, emissions here need not be a linear function of production. Let \( e_t = (1-\mu_t)h(y_t) \), where \( \mu_t \) is the fraction of emissions abated in period \( t \), and \( h \) is the function determining how emissions are related to output, holding constant abatement. Emissions could thus increase more rapidly than output if \( h \) is convex, or they could increase less rapidly if \( h \) is concave. The fraction of emissions abated \( \mu_t \) is determined by the amount of abatement \( z_t \) via the equation \( z_t/y_t = g(\mu_t) \); \( g \) thus relates the fraction of emissions abated to the fraction of output spent on abatement. The stock of capital evolves with a decay rate of \( \delta: k_t = (1-\delta)k_{t-1} + i_t \).

Finally, the technology shock evolves according to a Markov process, so that the probability distribution of \( a_{t+1} \) is a function of \( a_t \).

At the beginning of a period, \( k_{t-1} \) and \( x_{t-1} \) are already determined, and the realization of the productivity shock \( a_t \) occurs. Given those three state variables, the representative agent chooses abatement, consumption, and investment to maximize total expected discounted utility, subject to the constraints described above. This can be written as a dynamic programming problem in the following way.

\[
V(k_{t-1}, x_{t-1}, a_t) = \max_{z_t, i_t, c_t} [U(c_t, x_t) + \beta E_t V(k_t, x_t, a_{t+1})],
\]

such that

---

\(^{11}\) The productivity shock represents factors exogenous to producers that alter their production function. In addition to representing changes in the stock of knowledge or technology, it may also represent any changes to the legal or regulatory system that affect productivity (see Hansen and Prescott 1993).
\[ c_t + i_t + z_t \leq f(a_t k_{t-1}) \]
\[ k_t = (1 - \delta)k_{t-1} + i_t \]
\[ x_t = \eta x_{t-1} + e_t + e^{row} \]
\[ e_t = (1 - \mu_t)h(f(a_t k_{t-1})) \]
\[ z_t / f(a_t k_{t-1}) = g(\mu_t) \]

The operator \( E_t \) represents the expectation of future values of \( a_{t+1} \) at period \( t \), and the discount factor is \( \beta \). The problem can be rewritten, taking into account the constraints, as a choice over only the non-stochastic state variables. The inequality resource constraint must bind, which is ensured as long as preferences are non-satiated and the production and abatement functions are monotonically increasing. Then, the dynamic programming problem can be written as a choice over \( k_t \) and \( x_t \):

\[
V(k_{t-1}, x_{t-1}, a_t) = \max_{k_t, x_t} \left[ U(f(a_t k_{t-1}) - k_t + (1 - \delta)k_{t-1} - g(1 - \frac{x_t - \eta x_{t-1} - e^{row}}{h(f(a_t k_{t-1}))}) \cdot f(a_t k_{t-1}), x_t) + \beta E_t V(k_t, x_t, a_{t+1}) \right]
\]

In this format, first order conditions can be found for the choice of both \( k_t \) and \( x_t \). They are:

\[
0 = -U_c(c_t, x_t) + \beta E_t U_c(c_{t+1}, x_{t+1}) \cdot \{a_{t+1} f'(a_{t+1} k_t) + (1 - \delta)
\]
\[
- a_{t+1} f'(a_{t+1} k_t) \cdot [g'(\mu_{t+1}) f(a_{t+1} k_t) \cdot \frac{e_{t+1}}{h(f(a_{t+1} k_t)))^2}]h'(f(a_{t+1} k_t)) + g(\mu_{t+1})]\}
\]
\[
0 = U_c(c_t, x_t) \cdot g'(\mu_t) \cdot \frac{f(a_t k_{t-1})}{h(f(a_t k_{t-1}))}
\]
\[
+ U_x(c_t, x_t) - \beta E_t U_c(c_{t+1}, x_{t+1}) \cdot g'(\mu_{t+1}) \cdot \frac{\eta f(a_t k_{t-1})}{h(f(a_t k_{t-1}))}
\]

The first equation is the first order condition for the choice of \( k_t \), which is equivalent to the choice of investment this period \( i_t \). The first term, \(-U_c\), is the marginal cost of an additional unit of investment, which is the foregone marginal benefit of an additional unit of consumption. The second (long) term is the marginal benefit of an additional unit of investment. It is not realized until next period, so it is discounted and taken as an expectation. It equals the marginal benefit of an additional unit of consumption \( (U_c) \) times the marginal effect that investment has on next period's consumption. This marginal effect is composed of the three terms in the curly brackets in the second and third lines of the equation. First, the effect depends on the production
function and productivity shock, which indicates how more capital can lead to more consumption. Second, the capital depreciation rate $\delta$ determines how much of the stock value of capital remains, and thus the possible consumption in the next period. Third, it depends on the derivatives of the abatement function and the emissions function, since the chosen level of abatement next period affects how much of this period's investment is available for next period's consumption.

The second equation above is the first order condition for the choice of the pollution stock $x_t$. The first term, on the top line, is positive, and it is the marginal benefit from an additional unit of $x_t$. That benefit comes from the additional current-period consumption that is possible with higher $x_t$, due to the lower amount of abatement expenditure. The second and third terms are the marginal costs, appearing on the bottom line as written. The direct cost from increasing pollution and therefore reducing utility is $U_x$. Also, the additional unit of $x_t$ this period increases the amount of abatement necessary next period to achieve an equivalent level of emissions next period, thus reducing the budget for consumption next period. This effect is found in the third term of the equation.

These first order conditions, along with the constraints and the Markov process governing the productivity shocks, determine the paths of the endogenous variables. Without a further specification of the functional forms in the model, it is impossible to explicitly solve for these variables. I thus turn to calibration of this model, imposing functional forms and parameterizing them.

IV. Calibration

The sources for the calibration of the model fall into two main groups: macroeconomic parameters are taken from the RBC literature, and parameters related to emissions are taken from several studies that estimate the costs and benefits of emissions reductions. One additional parameter is estimated from the empirical results presented above. The model is calibrated to the US economy, and the pollutant considered is carbon dioxide, the leading greenhouse gas.

The macroeconomic parameters include a functional form and parameterization for the production function, the process governing the productivity shocks, the capital decay rate, and the discount factor. All of these parameters are commonly used in RBC papers, and it is to those papers that I turn for their calibration. Specifically, the values I use here are those used in recent
RBC literature, including Chang and Kim (2007), and pioneering papers, including Kydland and Prescott (1982).

The production function is \( f(ak) = (ak)^{\alpha} \), where \( 0 < \alpha < 1 \) to accommodate positive but diminishing marginal returns. In most RBC models, two inputs to production are present: capital and labor. Then, the production function is Cobb-Douglas: \( f(k,l) = ak^{\alpha}l^{1-\alpha} \). The capital share of income is taken as the value \( \alpha \). Here, production has only one input, since I am unconcerned with the labor market. In other words, the model here assumes fixed labor inputs and models only the response of capital inputs to production. The value used for \( \alpha \) is 0.36, capital's share in national income.

I use a quarter as the period of time. Given that period, capital depreciation \( \delta \) is found to be 2.5\%, and the discount factor \( \beta \) is 0.98627, consistent with a quarterly rate of return to capital of 1\%. This is equivalent to an annual discount rate of about 5\% and annual capital depreciation of about 9.6\%. Finally, the productivity shock evolved according to a Markov process with the following equation: \( \ln a_t = \rho \ln a_{t-1} + \varepsilon_t \), where the persistence parameter \( \rho \) is 0.95, and \( \varepsilon_t \) is an IID shock distributed normally, with mean zero and standard deviation \( \sigma_\varepsilon = 0.007 \).

Next I use chemistry and economics studies to calibrate the functions regarding emissions. The decay equation for the stock of pollution in the atmosphere, \( x_t = \eta x_{t-1} + e_t + e^{row}_t \), contains the parameter \( \eta \), which can be calibrated from the half life of atmospheric carbon dioxide. This value is not precisely known, and various papers use different estimates. In models with annual time periods, Falk and Mendelsohn (1993) and Nordhaus (1991) use a decay rate implying a half life of 139 years. The decay rate used in Reilly (1992) assumes a half life of 83 years. Moore and Braswell (1994) estimate the half life of atmospheric CO\(_2\) under a range of different assumptions, and consequently find a range of answers, from 19 to 92 years. For my base case, I use the value used in Reilly (1992) as a central value. In sensitivity analysis, I see how the value of the decay parameter affects the results. A half life of 83 years implies a quarterly parameter \( \eta \) of 0.9979. I assume that rest-of-world emissions \( e^{row}_t \) are maintained at a constant level \( e^{row} \). Thus, for simplicity international emissions do not respond to domestic business cycles. International business cycles, and therefore international emissions, are likely to be correlated with domestic business cycles (Backus et. al. 1995). Though domestic policy cannot control international emissions, it may be affected by the fact that those emissions are
correlated with domestic factor productivity. The US is responsible for about one-fourth of global anthropogenic carbon emissions, so $e^{row}$ is set at three times the steady-state value of $e$.

The damages from atmospheric carbon dioxide have been estimated in other papers, including in Nordhaus (1991, 2008), Bosello et al. (2006), and Stern (2008). These damages in this model are captured by the utility function $U(c,x_t)$. I assume for simplicity that consumption and emissions enter utility separably and with constant relative risk aversion. These assumptions imply a utility function of the form

$$U(c,x) = \sigma \frac{c^{1-\varphi_c}}{1-\varphi_c} - (1-\sigma) \frac{x^{1-\varphi_x}}{1-\varphi_x}.$$ 

The parameters $\varphi_c$ and $\varphi_x$ are the coefficients of relative risk aversion, and they are both set to equal 2 (Stern 2008, Weitzman 2007). The parameter $\sigma$ represents a weighting between consumption and pollution in utility, and is calibrated from the DICE-2007 model in Nordhaus (2008). For damages from CO$_2$, this utility is a reduced form function, since damages do not come directly from the pollution stock, but rather from temperature and other climate changes that are caused by the pollution stock. Nordhaus (2008) includes a complex carbon cycle model, where the mass of carbon in the atmosphere, the upper oceans, and the lower oceans dynamically evolve according to anthropogenic and natural emissions, and these carbon concentrations enter into a calibrated function determining atmospheric and ocean temperatures. Here, I abstract from those considerations and model the pollution stock directly in the utility function. This ignores some potentially important effects regarding the timing of abatement. However, as I will show in the simulation results below, deviations in emissions levels from business cycles have very little effect on the pollution stock because of the slow decay rate of CO$_2$. The damages from pollution do not change significantly with the business cycle and thus do not affect optimal abatement of CO$_2$; for this pollutant it is solely due to the cost side that optimal abatement varies. Adding an additional delay in the effect on utility of current-period emissions, such as would be the case if the evolution of temperature were modeled, would only further diminish the already negligible cyclical effects of emissions damages.

The weighting parameter $\sigma$ is calibrated in the following way. For a given level of the pollution stock, I use the equations in Nordhaus (2008) relating the atmospheric mass of carbon to evolving atmospheric and ocean temperatures to find the path of these temperatures over the extent of the period of simulation. Nordhaus (2008) also provides a function relating
atmospheric temperature to cost in terms of fraction of output lost.\textsuperscript{12} This represents global damages, but since my model considers optimal domestic policy I want US damages. I convert the global damage function using regionally disaggregated estimates for damages from temperature changes that are available in the model's documentation.\textsuperscript{13} Even when expressed as a fraction of output, domestic damages from temperature changes are less than global damages, a fact that is consistent with the EPA's Technical Support Document on benefits of greenhouse gas abatement.\textsuperscript{14} For a given level of the pollution stock (assumed to be constant for this calibration), I solve for the path of temperature and the associated domestic damages in each period of the simulation. I then discount those damages to come up with a measure of the current cost for a particular level of the pollution stock. This is done for 100 values of the pollution stock (ranging from 600 to 1200 GtC), and the results are shown in Figure 4. The y-axis plots the value of $\Omega$, which equals one minus the fraction of domestic output lost from climate change damage. It ranges from about one at a pollution stock level of 600 GtC (equivalent to about 280 ppm CO$_2$, the pre-industrial level), to about 0.987 at 1200 GtC (equivalent to about 580 ppm CO$_2$). The 2005 level is about 800 GtC or 380 ppm CO$_2$. The slope of this curve tells me how a change in the pollution stock affects output and thus utility from consumption. At the 2005 level of pollution, 800 GtC, the slope is $-1.8 \times 10^{-5}$, so a 10% increase in atmospheric carbon mass reduces $\Omega$ from 0.9974 to 0.9959. I use this slope to calibrate $\sigma$. Damages from increasing pollution by 10% over the steady state value (taken to be the 2005 level) equal $(1 - \sigma) \frac{(1.1x)^{-\varphi_c}}{1 - \varphi_x} - (1 - \sigma) \frac{x^{-\varphi_c}}{1 - \varphi_x}$. This should equal the loss in utility of decreasing from 0.9974 times steady state consumption to 0.9959 times steady state consumption, or $\sigma \frac{(0.9974c)^{1-\varphi_c}}{1 - \varphi_c} - \sigma \frac{(0.9959c)^{1-\varphi_c}}{1 - \varphi_c}$. This gives a calibrated value for $\sigma$ that depends on the steady state levels of pollution and consumption, and so $\sigma$ is calibrated along with solving for the steady state solution. In the base case, the weight on consumption is $\sigma = 0.0372$. This does not mean that utility from consumption is worth less than four percent of total

\textsuperscript{12} The model's equations are provided in Nordhaus (2008), and the parameter values are available on his website: http://nordhaus.econ.yale.edu/DICE2007.htm.

\textsuperscript{13} The documentation is available here: http://nordhaus.econ.yale.edu/Accom_Notes_100507.pdf. The relevant information is in the table on p. 24

\textsuperscript{14} The document can be found at www.regulations.gov searching for "Technical Support Document – Benefits." See Table 1 on p. 12.
utility, since the utility function includes the steady state values of consumption and utility, in arbitrary units. The steady state value of pollution is about 4000 while the steady state value of consumption is about 3. The small value of \( \sigma \) thus ensures that the damage function is calibrated to the simulation results shown in Figure 4.

The abatement cost function \( g(\mu_t) \) gives the cost, as a ratio of total output, of reducing the fraction \( \mu_t \) of baseline emissions. This function is taken directly from Nordhaus (2008). The function form used is \( g(\mu) = \theta_1 \mu^{\theta_2} \). The calibrated value of the exponent \( \theta_2 \) is 2.8, indicating a convex cost function. The coefficient, \( \theta_1 \), is actually a function of time in Nordhaus (2008), where each time period is ten years. I use the initial value (calibrated to 2005) of 0.05607, though this value decreases over time slightly (it drops to 0.0392 in 50 years).

Finally, the function mapping output to emissions controlling for abatement, \( h \), is estimated from the data described in section I. I impose an iso-elastic form to the function, so that \( e_t = (1-\mu_t)h(y_t) \) and \( h(y_t) = y_t^{1-\gamma} \). The parameter \( \gamma \) is calibrated based on an estimation of the log of emissions on the log of output, where the regression coefficient is \( 1-\gamma \). Regression results are presented in Tables 1 and 2. Table 1 uses monthly aggregate CO\(_2\) emissions and GDP and performs ARIMA regressions. The coefficient on the log of GDP ranges from 0.55 to 0.88. In Table 2, the data are annual but available at the state level. Column 2 includes state and year fixed effects and finds a coefficient of 0.67, while column 3 allows for an AR(1) error term and gets a coefficient of 0.34. For the base case value of \( \gamma \), I use the results from the initial ARIMA regression on the seasonally adjusted series from column 2 of Table 1, giving an exponent for \( h \) of \( 1-\gamma = 0.696 \). I will vary this parameter in sensitivity analysis.

Table 3 describes the parameters in the model, gives their calibrated base case values, and lists the source of each parameter calibration.

V. Model Solution and Simulation

A number of different solution methods are available to solve the parameterized model computationally. One method is value function iteration (VFI).\(^{15} \) Two disadvantages of that method are the need for discretization and the potential for computationally intensive (slow) solution methods. Instead, I solve the model by first log-linearizing about the steady state, and then analytically solving the system of linear rational expectations equations. This method is

\(^{15}\)See Adda and Cooper (2003).
advantageous for two reasons. First, the solution method is fast, taking less than one second on a typical PC compared to as long as several hours using VFI. Second, the fact that it is an analytic solution removes the need for discretizing or approximating. However, it comes as a cost: the non-linear model is approximated as a system of linear equations. In this application, the log-linearization should not present too much of a problem, since I am concerned with relatively small fluctuations about the steady state.

The specific solution method used for solving the system of linear equations is the Anderson-Moore algorithm (AMA), also referred to as AIM or the shooting method. Given a structural model consisting of a set of linear equations where the solution for period \( t \) can depend on the solutions for periods prior to and the expectations of periods subsequent to \( t \), the AMA provides a reduced form coefficient matrix, which gives the current period \( t \) variables as linear functions of only lagged values of the variables. Software for implementing the AMA is available; I use an application for Matlab provided by the Federal Reserve Bank of Boston. Anderson (2006) compares the practicality of several different methods for solving linear rational expectations models and finds that the AMA provides the highest accuracy and significant gains in computational efficiency.

Solving the model thus depends on a log-linearization of model presented above. This system of linear equations and the method for implementing the AMA on it are presented in Appendix A2. The Matlab code is available on the author’s website.

The solution provides a set of matrices that describes how the choice variables \((c_t, k_t, x_t)\) respond to different values of the state variables \((x_{t-1}, k_{t-1}, a_t)\), as well as different realizations of the innovation to productivity \((\varepsilon_t)\). This solution is difficult to interpret, but the results can be summarized graphically in two ways. First, one can examine impulse response functions: given an arbitrary non-zero value of \( \varepsilon_t \) in period \( t = 0 \) and no non-zero values in any other periods, what is the response path of the choice variables? Second, one can simulate a series of shocks \( \varepsilon_t \) and analyze the response of variables to those shocks. This simulates the actual business cycles in the economy and how policy can optimally respond.

\[\text{Anderson and Moore (1985).} \]
\[\text{17 The software and documentation are available here: } \text{http://www.bos.frb.org/economic/econbios/fuhrer/matlab.htm. Additionally, Zagaglia (2006) summarizes implementation of the AMA using Matlab.}\]
**Base Case Simulation**

Impulse response functions for the base case parameters are plotted in Figures 5 and 6. Both figures come from the same simulation of 100 periods, but they plot different variables (except for the productivity shock $a$, which is plotted in both figures). In the simulation, an innovation to the productivity shock $\varepsilon_t$ occurs in period 1. The size of this shock is equal to $\sigma_\varepsilon$, the standard deviation of the innovation in the calibrated model. Figure 5 plots the value of the productivity shock $a$ along with economic variables traditionally seen in RBC models: output $y$, capital stock $k$ and consumption $c$. Figure 6 repeats the plot the productivity shock $a$, while adding the response of three variables related to pollution: abatement $z$, single-period emissions $e$ and pollution stock $x$. The path of the productivity shock in both of these figures is exogenous; it decays at the rate $\rho = 0.95$. The value of output $y$ is also decreasing along with productivity. Output is not perfectly coincidental with productivity, since the choice between allocating resources between production and abatement is altered by the productivity shock. If the capital stock were kept fixed at the steady state value, then the $a$ and $y$ curves would be coincidental. Figure 5 shows, though, that capital is responding to the increased productivity. More is being invested, and thus capital is higher than it is in the steady state. Because capital is a stock good, its peak does not coincide with the productivity peak in period 1. Capital peaks around period 20 (year 5). Similarly, consumption is higher thanks to the increased productivity, but its peak occurs around period 15. The lag between the peak of productivity and the peak of consumption is not because consumption is a stock; it is not. Rather, it is a function of the resource constraint in each period, which is dependent on capital, a stock.

Figure 6 shows how optimal emissions policy responds to the productivity shock. The path of productivity is included again in Figure 6 for comparison. The increased productivity induces an increase in abatement expenditure $z$, which peaks at around period 10. Although more is being spent on abatement, the increase in output means that emissions will be higher for a fixed level of abatement. It is thus unclear in which direction emissions will go, but Figure 6 shows that emissions $e$ are higher after the productivity shock. This demonstrates the fact that, in the base case calibration of the model, the income effect is dominated by the price effect. Higher productivity yields more income, and the absence of pollution is a normal good, so less emissions will be demanded. On the other hand, abatement is costlier with high productivity, so the price effect causes more emissions to be demanded. Using the static model, I presented
conditions under which either effect might dominate. While the lack of an analytical solution to the dynamic model precludes analogous conditions, the simulation here identifies which effect dominates. Finally, note that the curve for pollution $x$ exhibits two striking features: it is of a much smaller magnitude than the other curves, and it is more persistent. Even after 100 periods it has just barely reached its peak and begun to fall. Both of these features are due to the fact that the decay rate for CO$_2$ is so low ($\eta = 0.9979$), so a change in any one year's emissions has little effect on the total stock, and deviations from the steady state of emissions take a long time to decay.

What do these impulse response functions say about how optimal policy responds to productivity shocks, for the base case given by these parameters? Three qualitative lessons can be learned. First, optimal emissions are procyclical; that is, periods of higher productivity are allowed higher emissions. Second, to a first order, a policy that pegs allowable emissions to GDP is optimal. Comparing the curve for $y$ in Figure 5 to the curve for $e$ in Figure 6, it appears that these two values are almost identical in shape. Third, the magnitudes of the adjustments to the pollution stock are quite small, because of the low decay rate of emissions.

Next, I simulate a draw of productivity shock innovations $\varepsilon_t$ from its calibrated distribution and see how the economy responds to that series of shocks. This is a simulation of an actual RBC economy, as opposed to the impulse response functions, which are abstractions designed to see the qualitative nature of responses. These simulations cannot explicitly show how individual policy variables respond to particular productivity shocks, but they can provide summary statistics of the nature of the business cycles under the optimal solution, such as the standard deviations of output and emissions, or the covariance of output and emissions.

Each simulation is of course dependent on the particular draw of shocks. Figure 7 presents the results from one such draw. The curve with no marking is output. After an initial 20 periods of fluctuations near zero, the economy experiences a long expansion followed by a long recession. Again, this is the result of this particular draw; another draw may yield a different qualitative pattern. The curve marked with squares is emissions; it closely follows output. This conforms with the impulse response functions, where the curves for $k$ and $e$ were

---

18 The income effect being dominated is unsurprising given that it comes from a temporary though persistent shock to income, which is small compared to permanent income. A two standard deviation positive shock to productivity increases total discounted output by less than one-hundredth of a percent, compared to total discounted output with no positive shock.
close to each other. The curve marked with circles in Figure 7 is the capital stock. The lag between the peaks of capital and the peaks of output is consistent with the lagged peak found for capital in the impulse response function; it is because capital is stock that accumulates over time. Finally, the curve marked with triangles is the pollution stock. It appears almost flat, though it actually fluctuates somewhat. As in the impulse response functions, its small magnitude is due to its very low decay rate.

In this particular simulation, the standard deviation of the cyclical component of output is 1.4%, almost the same as the corresponding value in the actual US economy (1.31%). The standard deviation of emissions is 0.97%, which is quite lower than in the US economy (2.04%). However, this simulation represents optimal emissions policy and therefore is not expected to match the data. The correlation coefficient between output and emissions is 0.9999, again conforming to the finding here: a policy that pegs emissions levels to output is a good first-order approximation of optimal policy.

Sensitivity Analysis

The results presented thus far are taken from simulations using the base case parameter values, listed in Table 3. Sensitivity analysis can be done by varying these parameters and seeing the effect on policy. One parameter that may be of interest is the decay rate of the pollution stock, \( \eta \). In the base case this value is 0.9979, meaning that CO\(_2\) stays in the atmosphere for a long time after it is emitted. How sensitive is the optimal policy to this parameter? Figure 8 suggests that the answer is not very sensitive. It plots the impulse response functions to the same one-standard-deviation shock to productivity as was simulated in Figures 5 and 6. It only presents the curve for \( e \), but it presents the curves for the solution under six different values of \( \eta \), going down all the way to zero, representing a pure flow pollutant like SO\(_2\) or other precursors of ozone like NO\(_x\). Surprisingly, the results are not very sensitive to this parameter. Even when pollution is purely a flow, the optimal emissions path is almost identical to the one in the base case, in which CO\(_2\) has a long half-life.\(^{19}\) Figure 9 shows the optimal impulse response functions for the pollution stock. Here, the differences are clear, and they are due to the different decay rates of the pollution stock.

\(^{19}\) Of course, the impulse response functions plotted are in proportional change from the steady-state value, and the steady-state values of the variables differ under each parameter set.
A key area of uncertainty regarding the effects of climate change is the magnitude of the damages it may create. The parameter $\sigma$, the weighting parameter on emissions damages in the utility function, is calibrated from estimates of proportional losses to world output arising from specified concentrations of atmospheric CO$_2$ as described in the previous section. However, these estimates are uncertain. Thus, I perform a sensitivity analysis on the weighting parameter $\sigma$. Figure 10 shows the impulse response functions for the optimal emissions levels at five different values of $\sigma$, including the steady state value of 0.0372. A lower value of $\sigma$ indicates that damages from pollution count more in social welfare. For most values of $\sigma$, including the steady state value and any value at least 0.01, the paths of emissions are quite similar. Despite the large variation in $\sigma$, emissions deviates from the steady state in about the same pattern for these values of $\sigma$. For these values, damages from pollution are a sufficiently small component of total utility that the price effect dominates the income effect. Emissions rise during high productivity periods because the price of abatement is too high. The optimal level of emissions, in response to an identical one-standard-deviation productivity shock, is always initially higher than in the steady state, but the deviation from the steady state has a smaller magnitude as $\sigma$ decreases.

However, for $\sigma = 0.001$, optimal emissions are initially higher than the steady state level but fall back to the steady state level faster than they do under simulations with higher values of $\sigma$. In fact, for this value of $\sigma$, emissions actually dip below the steady state value around period 30 and before gradually coming back to the steady state. Because the damages from emissions are higher, the income effect of demanding more abatement is relatively stronger than it is under the other values of $\sigma$. Thus, after period 30, the income effect begins to dominate the price effect. Since the effects of the positive productivity shock remain throughout this simulation, abatement is still more expensive than in the steady state. However, the pollution stock is higher than the steady state level, and under this simulation pollution is very harmful, so the income effect pushes for more abatement. The evolution of the pollution stock for the same five simulations is shown in Figure 11. The pollution stock begins to fall sooner with the lower values of $\sigma$, because emissions are lower in these simulations, with the largest drop-off coming from the lowest value of $\sigma$.

A third parameter considered in the sensitivity analysis is $\beta$, the social discount factor. This parameter is at the center of much debate over appropriate policy responses to climate
change, since damages occur far in the future (Stern 2008, Weitzman 2007, Nordhaus 2007). The question about the appropriate discounting of damages to future generations is somewhat tangential to the question at hand here; I am concerned with short-run business cycles and not long-run policy. Still, it is useful to consider the impact of the discount rate on the results found in this model. Figure 12 models impulse response functions of emissions, in deviations from the steady state value, to three different values for the discount factor in addition to the base case value of 0.98267. Figure 13 presents impulse response functions for abatement spending, again in deviations from the steady state value, for the same four values of $\beta$. With a lower discount factor $\beta$, society is less patient. When $\beta$ is 0.9, the relative value that society places on future periods compared to current periods is less than it is when $\beta$ is 0.99. Figure 12 shows that when $\beta$ is 0.9, emissions are higher in the first half of the 100-period simulation and lower in the second half than when $\beta$ is 0.99, in deviations from the steady-state values. This seems counterintuitive: one might think that a less patient society wants to push emissions farther into the future than a more patient society. However, Figure 13 shows that when $\beta$ is 0.9, abatement spending is higher in the first half of the simulation and lower in the second half than when $\beta$ is 0.99. The less patient society does in fact spend more on abatement in the near future and less in the far future than does a more patient society. The less patient society's emissions are higher than the more patient society's emissions in the first half of the simulation because output is also higher in this less patient society. In response to the same positive productivity shock, the less patient society (with $\beta = 0.9$) produces more in the near future than does the more patient society (with $\beta = 0.99$), increasing emissions in the near future despite higher abatement spending.

The last parameter upon which I perform sensitivity analysis is $\gamma$, the parameter that defines the elasticity of emissions with respect to output when abatement is held constant. In Figures 14 and 15, I vary $1 - \gamma$, the elasticity, from 0.25 to 1.2. The base case value of this elasticity was 0.696, but the estimate from Tables 1 and 2 varies across columns. Although all columns those tables find that emissions are inelastic, I allow $1 - \gamma$ to take the values of 1 and 1.2. Figure 14 plots the deviation from steady state in emissions resulting from a one standard deviation positive productivity shock. Higher values of $1 - \gamma$ imply a higher deviation from steady state of emissions in response to the shock. Even if the change in output was no different between the five simulations shown in Figure 14, these results are expected, since the parameter that is varied gives the response of emissions to changes in output. Figure 15 shows the change
in abatement spending between the five values of $1 - \gamma$. When the elasticity is higher, abatement spending is higher. Thus, though emissions increase more with a higher elasticity, optimal policy dampens this change by increasing abatement spending when emissions would be the highest. Optimal policy dampens the cyclicality of emissions at a higher rate the higher is the elasticity of emissions with respect to output.

All of the simulations come from a shock to total factor productivity, which creates both an income and a price effect as described earlier in the static model. This is a departure from much of the literature that models a reduced form abatement cost function with potential shocks to the slope of that function. In that case, when the value of the shock is known by both firms and regulators, the policy implications are clear since only a price effect arises: when abatement costs more, optimal policy is to abate less. This model can include shocks directly to abatement costs, and it can make those shocks autocorrelated, by including a shock term in the linearized equation for abatement costs ($z$). When such a shock is simulated in this model, the expected result is reached. During periods of high abatement cost, optimal policy is to abate less. However, the focus of this paper is to extend beyond a model with direct shocks to abatement, and to consider instead aggregate productivity shocks, where the direction of the response of optimal policy is not as clear.

VI. Decentralized Economy

The model presented and solved above represents a social planner's problem: the representative agent can choose investment, consumption, and emissions simultaneously to maximize total discounted utility. Thus, all externalities are internalized. If the goal is to find optimal environmental policy in response to correlated productivity shocks, then this model is helpful to find the levels of these variables that would maximize utility in the presence of these shocks. However, the model does not explicitly specify the particular policy that a government can take to achieve these first best results in the presence of the environmental externality. For this problem, I turn to an extension of the model where decision-making is decentralized among competitive firms, utility-maximizing individuals, and a benevolent government. I consider two types of government policies: an emissions tax on firms, and a quantity restriction on firms' emissions.
First consider the behavior of the representative firm. It seeks to maximize profits by choosing the appropriate level of capital and abatement. Its profit function is

\[ \pi_t = f(a, k_{t-1}) - \tau_t e_t - r_t k_{t-1} - z_t, \]

where \( \tau_t \) is the emissions tax, and \( r_t \) is the endogenously determined cost of capital. The price of output and the price of abatement are both normalized to one. The firms maximize profit subject to the emissions function \( e_t = (1 - \mu_t) h(f(a, k_{t-1})) \) and the abatement cost function \( z_t = f(a, k_{t-1}) g(\mu_t) \). In the event of a quantity policy, where firm emissions are fixed at the quantity \( q_t \), the firms are also subject to that constraint.

With a tax policy, the firm's profit maximizing behavior sets the marginal value product of capital equal to the rental rate and the marginal value product of abatement equal to its price, normalized to one. Thus,

\[ r_t = af'(a, k_{t-1}) [1 - \tau_t (1 - \mu_t) h'(f(a, k_{t-1})) - g(\mu_t)], \]

\[ \tau_t h(f(a, k_{t-1})) = f(a, k_{t-1}) g'(\mu_t). \]

On the other hand, a quantity policy does not have an explicit price for a unit of emissions, but the quantity restriction does create a shadow price.\(^{20}\) The constrained optimization problem is to maximize \( f(a, k_{t-1}) - r_t k_{t-1} - z_t \) such that \( (1 - \mu_t) h(f(a, k_{t-1})) = q_t \) and function \( z_t = f(a, k_{t-1}) g(\mu_t) \). Solving this problem using the Lagrangian method yields the following equations:

\[ r_t = af'(a, k_{t-1}) \{1 - g(\mu_t) - (1 - \mu_t) h'(f(a, k_{t-1})) \cdot f(a, k_{t-1}) g'(\mu_t) / h(f(a, k_{t-1}))\}, \]

\[ (1 - \mu_t) h(f(a, k_{t-1})) = q_t. \]

The second equation is just the quantity constraint. The first equation is analogous to the first equation under the tax policy, and it demonstrates that the shadow price on a unit of emissions created by the quantity constraint is \( f(a, k_{t-1}) g'(\mu_t) / h(f(a, k_{t-1})) \).

Next, consider the behavior of the representative consumer. The consumer is the owner of capital and rents it out at the market rate \( r_t \). The emissions policy, either the level of the emissions tax or the quantity, is determined exogenously and hence taken as given. Consumption and investment are chosen subject to a budget constraint

\[ c_t + i_t \leq r_t k_{t-1} + \tau_t e_t + \pi_t. \]

\(^{20}\) A tradable permit scheme creates an endogenous, explicit price of emissions. In this aggregate model with one representative firm, tradable permits are identical to a fixed quantity policy. The key difference between the tax and quantity policy is that, in the tax policy, the price is set by the government and the quantity of emissions determined endogenously, while in the quantity policy the price is set by the government and the price determined endogenously.
The consumer's income in each period comes from three sources: the rental income from the capital owned by the consumer and rented by the firm at rate \( r_t \), the emissions tax revenues collected by the government and returned to the consumer, and the firm profits, since the firm is owned by the consumer. Under the quantity constraint policy, no emissions tax revenues are earned and thus this term is absent, both from firm profits and from the consumer's budget constraint. The consumer does not directly spend anything on abatement; that decision is made by the firm and taken as given by the consumer. The consumer's maximization problem is

\[
\max_{\{c_t, k_t, x_t\}} \sum \beta^t U(c_t, x_t), \text{ such that }
\]

\[
c_t + i_t \leq r_t k_{t-1} + \tau_t e_t + \pi_t,
\]

\[
x_t = nx_{t-1} + e_t + e^\text{row} + e^\text{row},
\]

\[
k_t = (1-\delta)k_{t-1} + i_t
\]

along with the equations from the firm's decisions determining \( r_t, e_t, \) and \( \pi_t \). Thus, the consumer has no direct control over emissions; they are an externality.  

The consumer's problem can be expressed as choosing a path \( \{k_t\} \) to maximize total discounted utility. The consumer treats the policy variable as exogenous, whether it is a tax or a quantity constraint. The first order condition for the choice of \( k_t \) is

\[
-U^c(c_t, x_t) + \beta E[U^c(c_{t+1}, x_{t+1}) \cdot \frac{dr_{t+1}}{dk_t} + \frac{d\pi_{t+1}}{dk_t} + \frac{de_{t+1}}{dk_t} + (1-\delta)]
\]

\[
+ \beta E[U^c(c_{t+1}, x_{t+1}) \cdot \frac{dx_{t+1}}{dk_t} = 0
\]

The total derivatives \( \frac{dr_{t+1}}{dk_t}, \frac{d\pi_{t+1}}{dk_t}, \frac{de_{t+1}}{dk_t}, \) and \( \frac{dx_{t+1}}{dk_t} \) accommodate the fact that the consumer's choice of investment in this period, and hence capital available for next period, affects the equilibrium values of the capital rental rate, emissions, and profit chosen next period by the firm. Therefore, while emissions are not directly controlled by the consumer, their level is influenced by the consumer's investment decisions. Furthermore, in the quantity policy case, \( \tau_t = 0 \), and \( \frac{de_{t+1}}{dk_t} = \frac{dx_{t+1}}{dk_t} = 0 \), since the exogenous quantity policy precisely determines the level of emissions and hence emissions are unaffected by consumer investment choice. The

---

21 In this model with one consumer owning the one firm, the pollution externality may be internalized if the consumer controls the behavior of the firm and departs from profit-maximizing behavior because of pollution; see Gordon (2003). However, I assume that the externality is not internalized and that the firm continues to maximize profits.
expressions for the total derivatives that appear in the first order condition are given in the Appendix A3.

A benevolent government chooses the optimal value of the policy variable given the behavior of the firm and the consumer. In other words, the government chooses \( \{ \tau_t \} \) or \( \{ q_t \} \) to maximize total expected discounted utility, given the constraints from the firm's profit maximizing behavior and the constraints from the consumer's profit maximizing behavior.\(^{22}\) Because the government is modeled as benevolent, it has the same objective function as the consumer. Thus, the solution to the government's problem is equivalent to the solution to the problem where the consumer's decision and the government's decision are made simultaneously. This equivalence disappears when asymmetric information exists between consumers and government, a case that will be examined later. But here I take advantage of this simplification to ease the solution of the government's problem.

Under the tax policy, the action of the government is to choose the path of tax rates to maximize utility. This yields the following first order condition for \( \tau_t \):

\[
U_c(c_t, x_t) \cdot \left[ k_{t-1} \frac{dr_t}{d\tau_t} + \frac{d\pi_t}{d\tau_t} + e_t + \tau_t \frac{de_t}{d\tau_t} \right] + U_x(c_t, x_t) \frac{de_t}{d\tau_t} = 0.
\]

A marginal increase in the emissions tax has a marginal benefit of decreasing current period emissions, represented in the second term of the above equation. It also comes with a marginal cost, which is the foregone current period consumption, represented in the first term. These effects depend on the total derivatives \( dr_t/d\tau_t \), \( d\pi_t/d\tau_t \), and \( de_t/d\tau_t \), which are derived in Appendix A3.

Under the quantity policy, the government's first order condition for the choice of \( q_t \) is

\[
U_c(c_t, x_t) \cdot \left[ k_{t-1} \frac{dr_t}{dq_t} + \frac{d\pi_t}{dq_t} - \frac{dk_t}{dq_t} \right] + U_x(c_t, x_t) \frac{dx_t}{dq_t} = 0.
\]

As with the government's first order condition for the tax policy, this equation represents a tradeoff between the marginal benefit of reducing the emissions cap (the second term) and the marginal cost of reducing the cap (the first term). Appendix A3 solves for the terms in brackets.

---

\(^{22}\) The government is able to dynamically adapt the policy in light of new information on productivity shocks and state variables. This is thus a "feedback" policy, as defined in Hoel and Karp (2002). The alternative is an "open-loop" policy, where the government must choose the entire policy trajectory at the initial period. This other extreme, as well as other policy options along the spectrum between these two extremes, is considered later, when I allow for information asymmetry between regulators and firms/consumers.
From the pollution decay equation \( x_t = \eta x_{t-1} + e_t + e^{\text{row}_t} \) and the policy constraint \( e_t = q_t \), it is clear that \( dx/dq_t = 1 \).

The equilibrium under each policy contains eleven variables. The policy shock \( a_t \) is exogenous. The firm's first order conditions determine \( r_t, z_t, \) and \( \mu_t \). Consumption \( c_t \) is determined by the consumer's budget constraint, and firm profits \( \pi_t \) by the profit equation. The flow of emissions \( e_t \) is determined by the emissions equation \( g(z_t)f(a_t k_{t-1}) \), and the stock of pollution \( x_t \) by the expression \( \eta x_{t-1} + e_t + e^{\text{row}_t} \). The consumer's first order condition determines \( k_t \), and the government's first order condition determines the policy variables, either \( \tau_t \) or \( q_t \). Lastly, the production function determines \( y_t \). Thus, the eleven unknowns are determined by eleven equations, defining the equilibrium. The solution can be found computationally by log-linearizing the system and using the AMA.

Figures 16 and 17 present simulation results from decentralized economies where the government has the option of a quantity policy and a tax policy, respectively. In both simulations, the parameters are kept at the base case values. Both simulations use the same set of productivity shocks, but each economy is solved independently to allow the government to optimize based on the instrument available. Figure 16 shows the cyclical components of output \( y \) and the chosen emissions quota level \( q \) for a 100 period simulation. As in the centralized economy, the optimal level of emissions closely follows output. During expansions (such as the long one that occurs between about periods 20 and 60), the chosen emissions quota rises, because abatement is relatively more expensive when emissions are higher.

Figure 17 plots the cyclical component of output \( y \), the emissions tax rate \( \tau \), and the resulting level of emissions \( e \). Again, the optimal level of emissions closely follows output; emissions are procyclical. However, this is not achieved by reducing the emissions tax during expansions, as might be intuited. The emissions tax is also procyclical. How are higher emissions achieved in expansionary periods from higher emissions taxes? Because productivity shocks affect abatement costs, the marginal cost of abatement is higher during expansions. Also, the stock of capital responds positively to productivity shocks, and a higher level of capital increases baseline emissions. Without an increase in the emissions tax during expansions, emissions would increase more than optimally during these periods, and a countercyclical tax policy would exacerbate the problem. The simulations under the centralized economy show that optimal emissions policy is procyclical, with emissions rising during expansions. However, the
tax policy simulations show that in an important way the policy is countercyclical, since the emissions tax rate increasing during expansions means that emissions are lower than they otherwise would be during expansions.

The following four observations can be garnered from this simple extension of the model to a decentralized economy. First, with either a quantity policy or a tax policy, the government can achieve an emissions path that is equal to the first best path. Second, although emissions follow output, rising during expansions and falling during recessions, this is achieved with a tax rate that rises during expansions and falls during recessions. Because abatement is costlier during expansions, the emissions tax must rise to keep emissions from overshooting their optimal trajectory. Third, the volatilities of the tax and quantity policies are about equal. One may think that an emissions tax would have to vary less with productivity fluctuations than would an emissions quota, though this turns out to not be the case. Fourth and finally, a potentially interesting political economy observation can be made. During recessions it may be politically difficult or infeasible to get an emissions policy strengthened; consumers and producers will likely lobby against anything that may increase costs. If the policy is an emissions quota, then optimal policy is strengthened during recessions (the quota is reduced). However, if the policy is an emissions tax, it is weakened during recessions (the tax is reduced). Weakening policy during recessions may be more politically feasible than strengthening policy, perhaps suggesting an advantage of taxes over quotas in their ability to respond to productivity fluctuations.

These observations must be made with an important caveat. While information asymmetry between regulators and consumers or firms is the key component of the "prices versus quantities" literature pioneered by Weitzman (1974), the specification here has no such information asymmetry. Firms, consumers, and the government all face uncertainty about future values of productivity, but each agent’s information set is identical: they observe lagged values of the stock variables $k_{t-1}$ and $x_{t-1}$ and the current productivity shock $a_t$. To the extent that the simulation results here suggest that prices may be preferable to quotas due to the fact that the optimal tax drops during recessions and rises during expansions, this omits considerations of

---

23 Intuition might prescribe that, since the pollution stock is not varying by much, the marginal damages from emissions are not varying by much, and thus the emissions tax should not vary by much, compared to the quantity restriction. However, the government's first order condition for the choice of the emissions tax equates marginal costs with marginal benefits, where marginal costs are from foregone consumption. Since consumption is changing with the business cycle, the marginal cost of the emissions tax is also changing. It is this variance in consumption, not in the pollution stock, that leads to the variance in the emissions tax.
information asymmetries. Previous studies find that those considerations tend to support taxes over quotas as the optimal instrument for CO₂ (e.g. Newell and Pizer 2003); this study suggests an alternate reason for the same conclusion.

The model can be extended to consider the effects of information asymmetry on optimal policy in a decentralized economy. The economy with a tax policy and the one with a quantity policy each contain a first order condition for the government's choice of. This equation is based on the same information that consumers have at time \( t \), namely, all variables at time \( t-1 \) as well as the realization of the current period shock \( a_t \). If government has to set the tax or quota each period before it observes the shock then that first order condition will not hold. Instead, the government may choose a tax level based on whatever information set it has available. For example, the tax level chosen in period \( t \) may be a function of the value of the productivity shock observed in period \( t-1 \), or of the value of output in period \( t-1 \), or a combination of both.

In this case or limited information, the government's first order condition in the model is replaced by a linear equation relating the current emissions tax to the specified variables. The resulting system, if it is still linear in proportional deviations from steady state values, can be solved with AMA, and the dynamics can be evaluated.

Next I consider the results of some simulations of these models with information asymmetry, focusing on an economy with a tax policy. Figure 18 presents a simulated economy for 100 periods under a draw of productivity shocks. Each of the four panels presents a simulation under the same draw of shocks but for a different tax policy rule. The first panel presents the first best result, where the government can observe the current period productivity shock and set the tax accordingly (as in Figure 17). In all four panels for all simulation periods I plot the proportional deviation from steady state of the productivity shock \( a_t \), the emissions tax \( \tau_t \), expenditure on abatement \( z_t \), and the resulting fractional decrease in emissions \( \mu_t \). The first panel shows that the optimal tax rate is procyclical. During an expansion, such as the large one from periods 30-50, the emissions tax rises. As a result, expenditure on abatement \( z \) is also procyclical. The fraction of emissions controlled, \( \mu \), is also procyclical but with a much smaller magnitude.

In the next three panels I consider three different constrained policies under information asymmetry, where the government cannot observe the current period productivity shock. Instead, the government employs a simple linear rule for the emissions tax in each panel based
solely on the lagged value of output. In the first of these simulations (labeled "Constrained – A"), the emissions tax in period $t$ is set so that its proportional deviation from steady state is exactly equal to the proportional deviation of output in period $t-1$. For example, if the government observes that output is 1% greater than the steady state level in period $t-1$, it sets the period $t$ emissions tax equal to 1% higher than its steady state level. In this simulation, the tax is also procyclical, by construction. The volatility of the tax is higher than in the first best case; it rises more during expansions and falls more during recessions. As a result, the path of abatement expenditure is still procyclical and more volatile.

The simulation presented in the next panel ("Constrained – B") arises when the government's tax rule is exactly opposite to the one in the previous panel. Here, the emissions tax is set to equal the negative of lagged output in deviations from steady state. Thus, in the expansion from periods 30-50, the emissions tax falls below its steady state value. This countercyclical emissions policy induces countercyclical abatement; $z$ also falls during expansions and rises during recessions. The proportional magnitude of the change is about equal to the magnitude of the change in the tax rate. In contrast, for the previous constrained policy, the change in abatement was about twice the change in the tax rate. Thus, abatement is not a function only of the tax, but also of other variables in the economy. During periods with high productivity, abatement is relatively more costly than during periods of low productivity, since it diverts resources away from production. Yet demand for abatement and for clean air increase since the economy is richer. When the emissions tax rises during expansions, abatement increases at a rate even higher than the change in the emissions tax, due to this additional income effect. However, when the tax falls during expansions, as in the "B" simulation, abatement falls too, though by a lower magnitude, mitigated by the positive income effect.

Finally, the last panel of Figure 18 presents results from a simulation where the government's rule is simple: the emissions tax is static. Thus the curve in the graph for the proportional deviation from the steady state of the tax is constant at zero. Note that abatement increases during any expansions, from an income effect. However, the ratio of emissions abated, $\mu$, actually falls slightly during expansions. Why is abatement procyclical, while the fraction of emissions abated is countercyclical? It is because of the functional form of the abatement cost function: $z/y = g(\mu)$. The fraction of emissions abated is a function of the fraction of output $y$
spent on abatement \( z \). In the final panel of Figure 18, during expansions \( z \) rises, yet \( y \) rises as well, and it rises by more than does \( z \). Thus, \( \mu \) falls.

The simulations in Figure 18 explore how the economy responds to a suboptimal government policy that is constrained by information asymmetry. These are just three particular tax rules that the government can employ when their information set at period \( t \) is limited to information from \( t-1 \), such as lagged output. Other tax rules can be considered, including functions of other lagged variables or lags greater than one period. Figure 18 could also present curves for emissions or pollution stock under the other tax rules, as well as economic variables like consumption, investment, or capital stock. In fact, these variables change very little from the first best under the simulations with information asymmetry, and the resulting curves are almost indistinguishable. For example, even though abatement is countercyclical in simulation "B", emissions are actually procyclical, as they are in the first best simulation, and the magnitude of the changes is almost the same. Why do these other variables change so little under information asymmetry? The answer is that in the calibrated steady state, abatement spending is quite low relative to the budget constraint. The steady state fraction of emissions abated \( \mu \) is one-half of one percent. To maintain this level, only a tiny fraction of output must be spent on abatement. Thus, the changes in abatement and \( \mu \) shown in Figure 18, within ±5%, do not result in noticeable changes in emissions, nor do they affect the rest of the budget constraint. Hence consumption, investment, and capital stock do not vary by much from their steady state values under information asymmetry. Consequently, social welfare as a function of consumption and pollution does not vary by much, and in fact the variability in social welfare arising from productivity shocks dominates the variability between the first best and the information asymmetry policies.

The optimal tax or quantity policy under information asymmetry can thus not be found by maximizing realized social welfare under different shocks. Instead, I calculate the optimal policy by matching the impulse response functions of consumption and pollution (the two variables entering the utility function) for the first-best policy and the constrained policy. For example, in the case of a tax policy, suppose that the government is constrained to set the tax in period \( t \), in proportional deviation from its steady-state value, as a linear function of the deviation of output in period \( t-1 \). The policy is \( \tau_t = d y_{t-1} \), where \( d \) is a policy parameter representing how the tax should respond to lagged output. The optimal policy is found by
choosing the value of $d$ that minimizes the difference between the impulse response functions of consumption and pollution under the unconstrained and constrained scenarios.\textsuperscript{24} For the tax case, this value is 0.795; the optimal tax in period $t$ should be about 80\% of the deviation from steady-state output in period $t-1$. For the quantity policy, the value is 0.671. As in the optimal solution with no information asymmetry, both the optimal tax and the optimal policy are procyclical. The optimal quantity varies slightly less with the cycle than does the optimal tax policy.

VII. Conclusion

Economic fluctuations have real effects on policy, including environmental policy. During the 2000-2001 California electricity crisis, the Regional Clean Air Incentives Market (RECLAIM), a cap and trade scheme in Southern California for SO$_2$ and NO$_X$ emissions, was suspended. Emitters were effectively allowed to emit without paying any price. Recessions often motivate policy makers to enact significant policy changes, such as tax rebates. They may also avoid implementing costly policies; the failure of a prominent climate bill in the US Senate in June 2008 may be attributed in part to fears of increased energy costs in the face of an upcoming recession. Presumably, under a national greenhouse gas policy, the incentives will remain for policy makers to respond to ebbs and flows in the economy by altering the stringency of such a policy. How would policy optimally respond?

To answer that question, a dynamic stochastic general equilibrium business cycle model is calibrated to an economy that features damages from the stock pollutant carbon dioxide. A first-best solution to the social planner's problem finds that the optimal level of emissions increases with productivity. Thus, a quantity policy is relaxed during economic expansions and tightened during recessions. This result is attributed to an income effect, in which economic expansions create a higher demand for clean air, outweighing by a price effect, in which achieving a particular level of emissions is more costly during an economic expansion because of increased productivity. This result seems to be robust to various parameterizations of the model. Finally, a decentralized model is presented, where firms maximize profits, consumers maximize utility, and the government chooses a tax or quantity policy to maximize social welfare in the

\textsuperscript{24} I minimize simply the unweighted sum of deviations, in absolute value, between each period's value of the impulse response function for a simulation of 100 periods. Results are robust to the specification of the minimand, including a sum of squared differences.
presence of the environmental externality. This model is solved to see how asymmetric information affects the results of the model, as well as how it affects the choice between prices and quantities.

The model makes several simplifying assumptions that can be relaxed to answer other important questions. Because it contains only one representative agent, the model says nothing about distributional issues, either in the costs imposed by the policy or the benefits from cleaner air. Because it contains only one representative firm, the model cannot address the cost advantages of taxes or tradable permits over command and control policies in the presence of heterogeneous abatement costs. Labor is not included as an input to production, so the effect of business cycles on employment is not considered here, though that area has been studied extensively in the RBC literature. Behavioral anomalies are not modeled here; the representative agent is rational. This may be relevant to how policy could respond to business cycles. For example, if agents exhibit loss aversion, optimal policy during recessions may be impacted.\(^\_2^5\)

The model is solved by making linear approximations about the steady state. As the magnitude of business cycles increases, these linear approximations to deviations of the model's variables become worse, so that a solution method which does not require a linear model becomes more important. Lastly, the model presented is a quite basic DSGE business cycle model, where the only stochastic element is an autocorrelated productivity shock. Other elements sometimes included in DSGE models, including intermediate goods producers, Keynesian price dynamics, and monetary effects, are omitted.\(^\_2^6\)

However, this paper presents the first study of how productivity shocks affect optimal environmental policy in an RBC context. Dynamic considerations are important, and the extent of their importance partially depends on whether pollution is a stock or a flow. Federal regulation to mitigate climate change appears imminent, and considering the implications of business cycles on policy values is likely to be an important component of achieving efficiency.

References

\(^\_2^5\) To address a similar concern, Fujii and Karp (2008) develop a method for computing optimal long-run climate policy paths under a non-constant pure rate of time preference.

\(^\_2^6\) See Justiniano and Primiceri (2008) for a model including these features. Gali (1999, 2004) argues that technology shocks play only a limited role, if any, in explaining business cycles, though this is disputed by Christiano et. al. (2003).


**Appendix**

**A1: Comparative Statics of Static Model**

Applying the implicit function theorem to the first order condition of the static model presented in section 2, after setting the production function to be linear \( f(ak) = ak \), the effect of a change in the income shock \( b \) on capital can be written as

\[
\frac{dk}{db} = \frac{-d''g'ak(-g'k + g) - d'(-kg' + g')}{d''a(-g'k + g)^2 + d'(kg' - 2g')}
\]

In this expression, \( d' \) is shorthand for \( d'(e) \) evaluated at the equilibrium, and likewise for first and second derivatives of \( g \). In the denominator, all of the terms are positive definite. The numerator is also positive definite, thus indicating that an increase in \( b \) leads to an increase in \( k \). The effect on abatement \( z \) can be solved by taking advantage of the resource constraint: \( k + z = b \). This implies that \( dz/db = 1 - dk/db \). Using the expression for \( dk/db \) in this expression for \( dz/db \), it can be shown that

\[
\frac{dz}{db} = \frac{d''(-g'k + g)ag - d'g'}{d''a(-g'k + g)^2 + d'(kg' - 2g')}
\]
Again, the numerator and the denominator are both positive, leading to the result in the text that an increase in \( b \) leads to an increase in both \( k \) and \( z \).

The effect on emissions is found from the emissions equation \( e = f(ak)g(z) \), so that \( de/db = g'(z)f(ak)dz/db + g(z)af'(ak)dk/db \). Substituting in the expressions for \( dz/db \) and \( dk/db \) and setting \( f(ak) = ak \) yields:

\[
\frac{de}{db} = -d'ag^2 k - gkg'' + gg' \cdot
d''a(-g'k + g)^2 + d'(kg' - 2g').
\]

All terms in the numerator are negative. Thus, emissions decreases with an increase in income.

The response of optimal policy to a change in the exogenous productivity shock \( a \) can also be found using the implicit function theorem:

\[
\frac{dk}{da} = \frac{1 - (-g'k + g)(d'' + gka)}{d''a(-g'k + g)^2 + d'(g''k - 2g')}
\]

The denominator is just \( a \) times the denominator from the earlier three expressions and therefore positive, and the numerator can be broken up into two parts which all can be signed, as written in the text. The resource constraint \( k + z = b \) implies that \( dk/da = -dz/db \). Finally, the effect on emissions can be found from taking the derivative of the emissions equation: \( de/da = af'(ak)g(z)dk/da + f(ak)g'(z)dz/da \). Substituting in the appropriate expressions and simplifying terms yields

\[
\frac{de}{da} = -g'k + g + d'gk^2 g'' - d'(g^2 + g'^2 k^2)
\]

\[
d''a(-g'k + g)^2 + d'(g''k - 2g').
\]

A2: Linearizing and Solving the Dynamic Model

The equations describing the model as presented in Section 3 and parameterized in Section 4 are the following:

\[
\ln a_t = \rho \ln a_{t-1} + \epsilon_t
\]

\[
x_t = \eta x_{t-1} + \epsilon_t + e^{row}_t
\]

\[
c_t = (a_t k_{t-1})^\alpha - k_t + (1-\delta) k_{t-1} - \theta_t \mu_t^{\delta_t} (a_t k_{t-1})^\alpha
\]

\[
\sigma c_t^{-\phi_t} \theta_t \theta_1 \mu_t^{\delta_t-1} (a_t k_{t-1})^{\alpha \gamma (1-\gamma)} - \frac{(1-\sigma)}{c_t^{\phi_t}} - \beta \sigma c_{t+1}^{-\phi_t} \theta_t \theta_2 \mu_{t+1}^{\delta_t-1} (a_{t+1} k_t)^{\alpha \gamma (1-\gamma)} = 0
\]

\[
-\frac{\sigma}{c_t^{\phi_t}} + \frac{\beta \sigma}{c_{t+1}^{\phi_t}} \{ \alpha a_{t+1} k_t^{\alpha -1} [1 - \theta_t \theta_2 \mu_t^{\delta_t-1} (1-\gamma)(1-\mu_{t+1}) - \theta_t \mu_t^{\delta_t}] + (1-\delta) \} = 0
\]
\[ e_t = (1 - \mu_t)(a_t k_{t-1})^{\alpha(1-\gamma)} \]

The first equation described the Markov process governing the productivity shock; the second equation describes the evolution of the pollution stock; the third equation is the budget constraint, where expenditure on abatement \( z_t \) is replaced with the parameterized value of \( \theta_t \mu_t^{\theta_t} (a_t k_{t-1})^\alpha \). The next two equations are the parameterized and simplified versions of the first order conditions for the choice of pollution \( x_t \) and capital \( k_t \), and the final equation is the emissions function.

The model is solved by linearizing about the steady state, so first a steady state solution must be found. This is done by dropping the time subscripts from each of the five equations governing the dynamic model and solving. Additionally, the steady state value for the productivity shock is set to 1, and the steady state value of rest-of-world emissions \( e^{row} \) is set to three times the value of domestic emissions, so that the steady state version of the second equation is \( x = \eta x + e + 3e \). An analytical solution for the remaining steady state values cannot be found, but the system of equations can be reduced to just one equation in one unknown: \( \mu \). In the fifth equation, \( c \) drops out in the steady state, so it can be used to solve for \( k \) as a function of \( \mu \). Plugging this into the third equation gives \( c \) as a function of \( \mu \), and into the second equation gives \( x \) as a function of \( \mu \). Finally, all of these expressions can be plugged into the fourth equation, which can be solved numerically for \( \mu \).

The six equations can then be log-linearized about the steady state values of the six variables. Using a tilde to denote a proportional deviation from the steady state value of a variable, and a bar to denote the steady state value itself, the six linearized equations are

\[ \tilde{\alpha}_t = \rho \tilde{\alpha}_{t-1} + \varepsilon_t \]
\[ \tilde{x}_t - \eta \tilde{x}_{t-1} - (1 - \eta)\tilde{e}_t = 0 \]
\[ \tilde{c}\tilde{c}_t = \bar{k}^\nu (\alpha \tilde{\alpha}_t + \alpha \tilde{k}_{t-1}) + \bar{k}\tilde{k}_t - (1 - \delta)\bar{k}\tilde{k}_{t-1} + \theta_t \bar{\mu}^{\theta_t} \bar{k}^\alpha \left( \theta_2 \tilde{\mu}_t + \alpha \tilde{\alpha}_t + \alpha \tilde{k}_{t-1} \right) = 0 \]
\[ \frac{\sigma}{\bar{\varepsilon}^{\vartheta} \varpi} \theta_t \theta_2 \bar{\mu}^{\theta_t - 1} \bar{k}^{\alpha \gamma} (1 - \gamma) [ - \varphi_c \tilde{c}_t + (\theta_2 - 1) \tilde{\mu}_t + \alpha \tilde{\alpha}_t + \alpha \tilde{k}_{t-1} ] - \frac{(1 - \sigma)}{\bar{x}^{\vartheta}} ( - \varphi_c \tilde{x}_t ) \]
\[ - \frac{\beta \sigma \eta}{\bar{\varepsilon}^{\vartheta} \varpi} \theta_t \theta_2 \bar{\mu}^{\theta_t - 1} \bar{k}^{\alpha \gamma} (1 - \gamma) [ - \varphi_c \tilde{c}_{t+1} + (\theta_2 - 1) \tilde{\mu}_{t+1} + \alpha \tilde{\alpha}_{t+1} + \alpha \tilde{k}_{t+1} ] = 0 \]
\[
\frac{\sigma}{c} \varphi \bar{c} + \frac{\beta \sigma \alpha k^{\alpha-1}}{c} \left[ -\varphi \bar{c}_{t+1} + \alpha \bar{a}_{t+1} + (\alpha - 1) \bar{k}_t \right] \\
- \frac{\beta \sigma \alpha}{c} \bar{k}^{\alpha-1} \left[ \theta \theta z \mu^{\alpha-1} \right] \left[ 1 - \gamma \right] \left[ \alpha \bar{a}_{t+1} + (\alpha - 1) \bar{k}_t + (\theta - 1) \bar{\mu}_{t+1} - \varphi \bar{c}_{t+1} \right] \\
+ \frac{\beta \sigma \alpha}{c} \bar{k}^{\alpha-1} \left[ \theta \theta z \mu^{\alpha-1} \right] \left[ 1 - \gamma \right] \left[ \alpha \bar{a}_{t+1} + (\alpha - 1) \bar{k}_t + \theta \bar{\mu}_{t+1} - \varphi \bar{c}_{t+1} \right] \\
+ \frac{\beta \sigma (1 - \delta)}{c} \left( -\varphi \bar{c}_{t+1} \right) = 0
\]

\[
\bar{e}_t - \bar{k}^{\alpha (1 - \gamma)} \left[ \alpha (1 - \gamma) \bar{a}_t + \alpha (1 - \gamma) \bar{k}_{t-1} \right] + \mu \bar{k}^{\alpha (1 - \gamma)} \left[ \bar{\mu}_t + \alpha (1 - \gamma) \bar{a}_t + \alpha (1 - \gamma) \bar{k}_{t-1} \right] = 0
\]

This set of linear equations can be solved by the AMA, which is suited for solving linear rational expectations models. The model is a rational expectations model in the sense that all of the variables dated \( t+1 \) are actually expectations of those values, though the expectations operator is dropped in the equations written above. The structure of these models, as described by Zagaglia (2005), is

\[
\sum_{i=-\tau}^{0} G_i x_{t+i} + \sum_{i=1}^{\theta} F_i E_i x_{t+i} = e_t .
\]

The constants \( \tau > 0 \) and \( \theta > 0 \) are the number of lags and leads, respectively. In this model, both of these values are equal to 1. The matrix \( G_i \) is the coefficient matrix on the lagged and contemporaneous values of the variables \( x \), and \( F_i \) is the coefficients on the future values. The shock is represented as \( e_t \) and has expectation zero. Taking expectations and simplifying the above equation yields

\[
\sum_{i=-\tau}^{\theta} H_i E_i x_{t+i} = 0 ,
\]

where \( H_i \) are referred to as the structural coefficient matrices. These matrices are input into the matlab code for AMA, and a set of solution matrices is output. The solution can be used to find the reduced form of the structural model:

\[
x_t = \sum_{i=-\tau}^{\theta} B_i x_{i+t} + B_0 e_t .
\]

The matrices \( B_i \) and \( B_0 \) are used to find impulse response functions to a technology shock \( (\bar{e}_t) \), or to simulate cycles. The matlab code is available upon request from the author.
A3: First Order Conditions in Decentralized Model

In the first order condition for the consumer’s problem in the decentralized model, four total derivatives appear: \( dr_{t+1}/dk_t \), \( de_{t+1}/dk_t \), \( d\pi_{t+1}/dk_t \), and \( dx_{t+1}/dk_t \). They reflect the fact that the consumer’s investment choice affects the outcome variables indirectly through equilibrium decisions by the firm. These derivatives can be different for a tax policy and for a quantity policy, so I consider each in turn.

The expression in brackets in the consumer’s first order condition containing several derivatives is equal to \( dc_{t+1}/dk_t \):

\[
 r_{t+1} + k_t \frac{dr_{t+1}}{dk_t} + \frac{d\pi_{t+1}}{dk_t} + \tau_{t+1} \frac{de_{t+1}}{dk_t} + (1 - \delta) .
\]

This can be simplified by solving for \( d\pi_{t+1}/dk_t \). Since firm expenses for capital and emissions taxes are given back to the consumer, the entire expression above is simplified to

\[
a_{t+1} f'(a_{t+1} k_t) - \frac{dz_{t+1}}{dk_t} + (1 - \delta) .
\]

The only derivative that needs to be found in this expression is thus \( dz_{t+1}/dk_t \). This derivative can be evaluated using the abatement cost function \( z_{t+1} = (a_{t+1} k_t)^\alpha \theta \mu_{t+1}^{\theta 2} \), yielding \( dz_{t+1}/dk_t = z_{t+1} [a/k_t + (\theta 2/\mu_{t+1})](d\mu_{t+1}/dk_t) \). This is in terms of another derivative, \( d\mu_{t+1}/dk_t \), which can be solved for using the implicit function theorem on the firm’s first order condition for choice of abatement: \( \tau h(f(a_{t+1} k_t)) = f(a_{t+1} k_t) g'(\mu_t) \). Doing so yields \( d\mu_{t+1}/dk_t = \alpha \gamma \mu_{t+1}/(\theta 2 - 1) k_t \), and substituting this into the earlier expression gives \( dz_{t+1}/dk_t = \alpha z_{t+1} (\theta 2 (1 + \gamma) - 1)/[k_t (\theta 2 - 1)] \). Thus, the complete expression in the brackets of the consumer’s first order condition is

\[
\frac{dc_{t+1}}{dk_t} = \frac{\alpha \gamma}{k_t} + (1 - \delta) - \frac{\alpha z_{t+1}}{k_t (\theta 2 - 1)} (\theta 2 (1 + \gamma) - 1) .
\]

The derivative \( dx_{t+1}/dk_t \) can be found by noting that the equation \( x_t = \eta x_{t-1} + e_t + e^{row}_t \) and the assumption that \( e^{row} \) is constant imply that \( dx_{t+1}/dk_t = dx_{t+1}/dk_t \). This can be taken from the equation for emissions, \( e_{t+1} = (1 - \mu_{t+1}) \cdot h(f(a_{t+1}, k_t)) \). Again, the derivative can be simplified under the parameterizations of the production and abatement functions, yielding

\[
\frac{dx_{t+1}}{dk_t} = \frac{\alpha \gamma}{k_t} [(1 - \gamma) (1 - \mu_{t+1}) (1 - \gamma) - \frac{\gamma \mu_{t+1}}{\theta 2 - 1}] .
\]

Next, consider the values of these derivatives in the consumer’s first order condition in the context of the quantity policy. As mentioned in the text, \( de_{t+1}/dk_t = dx_{t+1}/dk_t = 0 \) under the
quantity policy, since emissions are determined exogenously by the government and is independent of investment. The expression for $dc_{t+1}/dk_t$ can again be simplified by substituting in the profit function derivative, resulting again in the expression $a_{t+1}f'(a_{t+1}k_t) - dz_{t+1}/dk_t + (1 - \delta)$.

However, in the quantity policy, $dz_{t+1}/dk_t$ is different than it is in the tax policy. It is in terms of $d\mu_{t+1}/dk_t$, which can be solved for using the implicit function theorem on the firm's quantity constraint. Doing so and substituting in the resulting expressions yields

$$\frac{dc_{t+1}}{dk_t} = \frac{\alpha y_{t+1}}{k_t} + (1 - \delta) - \frac{\alpha z_{t+1}}{k_t} (1 - \theta_2 (1 - \gamma) + \frac{\theta_2 (1 - \gamma)}{\mu_{t+1}})$$

In the government's first order condition for the choice of $\tau_t$, three derivatives appear: $dr_t/d\tau_t$, $d\pi_t/d\tau_t$, and $de_t/d\tau_t$. As in the previous first order condition, the expression in brackets can be simplified by substituting in the derivative of the expression for profits. After doing so, the expression for $dc_t/d\tau_t$ simplifies to just $-dz_t/d\tau_t$. This derivative can be found from the abatement cost function, and contains the derivative $d\mu_t/d\tau_t$, which can be found by using the implicit function theorem on the firm's first order condition relating abatement to the tax level: $\tau_t h(f(a_t k_t)) = f(a_t k_t) g'(\mu_t)$. After substituting in all evaluated expression, the derivative $dc_t/d\tau_t$ simplifies to $-\theta z_t (\theta_2 - 1)$. The last derivative can be evaluated from the emissions equation:

$$\frac{de_t}{d\tau_t} = -h(f(a_t k_t)) \frac{d\mu_t}{d\tau_t} = -y_t (1 - \gamma) \frac{\mu_t}{\tau_t (\theta_2 - 1)}.$$

Finally, the government's first order condition for the choice of $q_t$ contains the derivatives $dr_t/dq_t$, $d\pi_t/dq_t$, and $dk_t/dq_t$. This last derivative is equal to zero, since in solving the government's first order condition, the consumer's choice of $k_t$ is held fixed. The rest of the expression in brackets can be simplified by solving for and substituting in $d\pi_t/dq_t$. After doing so, the entire expression in brackets reduces to $-dz_t/dq_t$, which itself can be shown to equal $\theta \mu_t y_t (1 - \gamma)$.

The set of equations can be solved for steady state values, linearized, and input into AMA code to find a solution, as described in the last appendix section. Code is available from the author by request.
Note: Data are from Blasing et. al. (2004) and represent total carbon emissions from fossil fuel combustion at the monthly level. One teragram = $10^{12}$ grams.
Note: Emissions data are from Blasing et. al. (2004) and represent total carbon emissions from fossil fuel combustion at the monthly level. They are seasonally adjusted using the X-12-ARIMA program. GDP data are from the Bureau of Economic Analysis and are seasonally adjusted. Both series are normalized to the January 1981 levels.
Figure 3

Note: The values are the cyclical residuals from applying the HP filter to quarterly carbon emissions and GDP data.
Figure 4

Notes: Y-axis values are simulated losses in output from different atmospheric concentrations of carbon based on Nordhaus (2008).
Figure 5

Impulse Response Functions - A

Proportional Deviation

- $a$ - productivity
- $y$ - output
- $k$ - capital
- $c$ - consumption
Figure 6

Impulse Response Functions - B

Period

Proportional Deviation

\( x \times 10^{-3} \)

Legend:
- \( a \) - productivity
- \( z \) - abatement
- \( e \) - emissions
- \( x \) - pollution
Figure 7

Business Cycle Simulation

- Green line: y - output
- Yellow line: e - emissions
- Red line: k - capital
- Gray line: x - pollution

Proportional Deviation vs. Period

Period: 0 to 100
Proportional Deviation: -0.025 to 0.025
Figure 8

Sensitivity Analysis - Impulse Response Function

- Proportional Deviation of Emissions
- Sensitivity Analysis
- Impulse Response Function

- $\eta = 0$
- $\eta = 0.2$
- $\eta = 0.4$
- $\eta = 0.6$
- $\eta = 0.8$
- Base case $\eta = 0.9979$

Graph showing the proportional deviation of emissions over time for different values of $\eta$. The graph includes a range of values for $\eta$ and a base case value, illustrating the impact on emissions over a period of 100 time units.
Figure 9

Sensitivity Analysis - Impulse Response Function

Proportional Deviation of Pollution Stock

- \eta = 0
- \eta = 0.2
- \eta = 0.4
- \eta = 0.6
- \eta = 0.8
- base case \eta = 0.9979
Figure 10

Sensitivity Analysis - Impulse Response Function

Proportional Deviation of Emissions

\( x \times 10^{-4} \)

Period

Proportional Deviation of Emissions

\( \sigma = 0.001 \)
\( \sigma = 0.01 \)
\( \text{base case } \sigma = 0.0372 \)
\( \sigma = 0.25 \)
\( \sigma = 0.99 \)
Figure 11

Sensitivity Analysis - Impulse Response Function

- \( \sigma = 0.001 \)
- \( \sigma = 0.01 \)
- base case \( \sigma = 0.0372 \)
- \( \sigma = 0.25 \)
- \( \sigma = 0.99 \)
Figure 12

Sensitivity Analysis - Impulse Response Function

Proportional Deviation of Emissions

- $\beta = 0.9$
- $\beta = 0.95$
- Base case $\beta = 0.98267$
- $\beta = 0.99$
Figure 13

Sensitivity Analysis - Impulse Response Function

- Proportional Deviation of Abatement
- Period

Graph showing sensitivity analysis with different values of beta:
- Beta = 0.9
- Beta = 0.95
- Base case beta = 0.98267
- Beta = 0.99
Figure 14

Sensitivity Analysis - Impulse Response Function

- 1-gamma = 1.2
- 1-gamma = 1
- Base case 1-gamma = 0.696
- 1-gamma = 0.5
- 1-gamma = 0.25
Figure 15

Sensitivity Analysis - Impulse Response Function

- $1 - \gamma = 1.2$
- $1 - \gamma = 1$
- Base case $1 - \gamma = 0.696$
- $1 - \gamma = 0.5$
- $1 - \gamma = 0.25$

Proportional Deviation of Abatement

Period

$10^{-3}$
Figure 16

Decentralized Economy - Quantity Policy

Proportional Deviation

Period

Proportional Deviation vs Period graph for Decentralized Economy - Quantity Policy.
Note: The $x$-axis marks the simulation period and the $y$-axis marks the proportional deviation from steady state values. Each panel is a separate simulation under the same set of productivity shocks. The first panel is the first best solution. The second panel ("Constrained – A") is a constrained policy where the emissions tax deviation in period $t$ is fixed to equal the output deviation in period $t-1$. The third panel ("Constrained – B") is a constrained policy where the emissions tax deviation in period $t$ is fixed to equal the negative of the output deviation in period $t-1$. The final panel is a constrained policy where the emissions tax is kept constant.
Table 1

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnco2emis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lnco2sa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP_lnco2sa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BK_lnco2sa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF_lnco2sa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BW_lnco2sa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DS12.lngdp</td>
<td>0.758***</td>
<td>(0.175)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.lngdp</td>
<td>0.696***</td>
<td>(0.181)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>5.42e-05</td>
<td>-0.000866*</td>
<td>(6.39e-05)</td>
<td>(0.000507)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP_lngdp</td>
<td></td>
<td>0.859***</td>
<td>(0.126)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BK_lngdp</td>
<td></td>
<td></td>
<td>0.639***</td>
<td>(0.147)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF_lngdp</td>
<td></td>
<td></td>
<td></td>
<td>0.545***</td>
<td>(0.00611)</td>
<td></td>
</tr>
<tr>
<td>BW_lngdp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.723**</td>
<td>(0.280)</td>
</tr>
<tr>
<td>Observations</td>
<td>263</td>
<td>275</td>
<td>276</td>
<td>180</td>
<td>276</td>
<td>276</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. Column 1 presents results from a seasonal ARIMA(1,1,1)×(0,1,1)_{12} regression of log of CO₂ (not seasonally adjusted) on log of GDP. Column 2 presents results from an ARIMA(1,1,2) regression of log of CO₂ (seasonally adjusted) on log of GDP. AR, MA, and cointegrating equation terms are not reported for either regression. Columns 3-6 present results from least squares regression of detrended emissions on detrended output allowing for Newey-West standard errors, where the series are detrended by either Hodrick-Prescott (Column 3), Baxter-King (Column 4), Christiano-Fitzgerald (Column 5), or Butterworth (Column 6) filters. Optimal lag lengths are all determined by minimizing Akaike Information Criterion. Constant term is omitted from regressions using detrended series. *** p<0.01, ** p<0.05, * p<0.1
### Table 2

<table>
<thead>
<tr>
<th>State-Level Annual CO₂ Emissions and GDP</th>
<th>(1) Inemis</th>
<th>(2) Inemis</th>
<th>(3) Inemis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ingdp</strong></td>
<td>0.874***</td>
<td>0.666***</td>
<td>0.336***</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.128)</td>
<td>(0.029)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>-7.038***</td>
<td>-4.182***</td>
<td>-0.835***</td>
</tr>
<tr>
<td></td>
<td>(0.67)</td>
<td>(1.35)</td>
<td>(0.015)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>1989</td>
<td>1989</td>
<td>1938</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.77</td>
<td>0.98</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>State fixed effects?</strong></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Year fixed effects?</strong></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>AR(1) error term?</strong></td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: Standard errors are in parentheses. Standard errors are clustered at the state level in columns 1 and 2. *** p<0.01, ** p<0.05, * p<0.1

Data are logs of annual state-level carbon emissions and GDP.

### Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.36</td>
<td>Curvature of production function: (f(ak) = (ak)^{\alpha})</td>
<td>Chang and Kim (2007), Kydland and Prescott (1982)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.98267</td>
<td>Quarterly discount rate</td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.025</td>
<td>Quarterly capital depreciation</td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.95</td>
<td>Persistence of productivity shock</td>
<td></td>
</tr>
<tr>
<td>(\sigma_e)</td>
<td>0.007</td>
<td>Standard deviation of IID productivity innovation</td>
<td></td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.9979</td>
<td>Pollution decay</td>
<td>Reilly (1992)</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>0.05607</td>
<td>Abatement cost function: (g(\mu) = \theta_1\mu^{\theta_2})</td>
<td>Nordhaus (2008)</td>
</tr>
<tr>
<td>(\theta_2)</td>
<td>2.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.0372</td>
<td>Weight on consumption in utility function</td>
<td></td>
</tr>
<tr>
<td>(\varphi_e, \varphi_x)</td>
<td>2</td>
<td>Coefficient of relative risk aversion</td>
<td>Stern (2008), Weitzman (2007)</td>
</tr>
<tr>
<td>(1-\gamma)</td>
<td>0.696</td>
<td>Elasticity of emissions with respect to output</td>
<td>Estimated from monthly emissions and GDP data, see Table 1</td>
</tr>
</tbody>
</table>